Dynamics and long term evolution of the space debris

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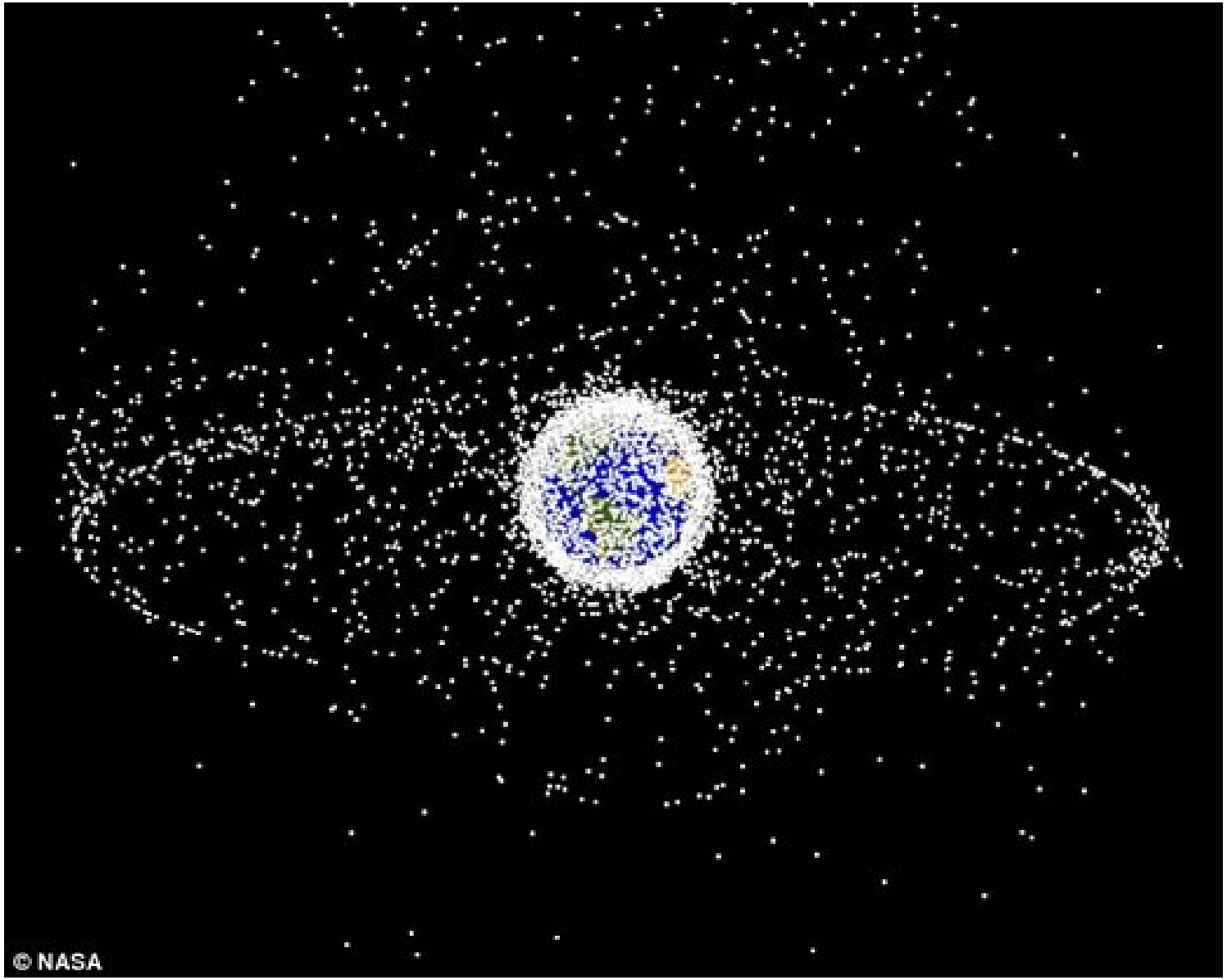
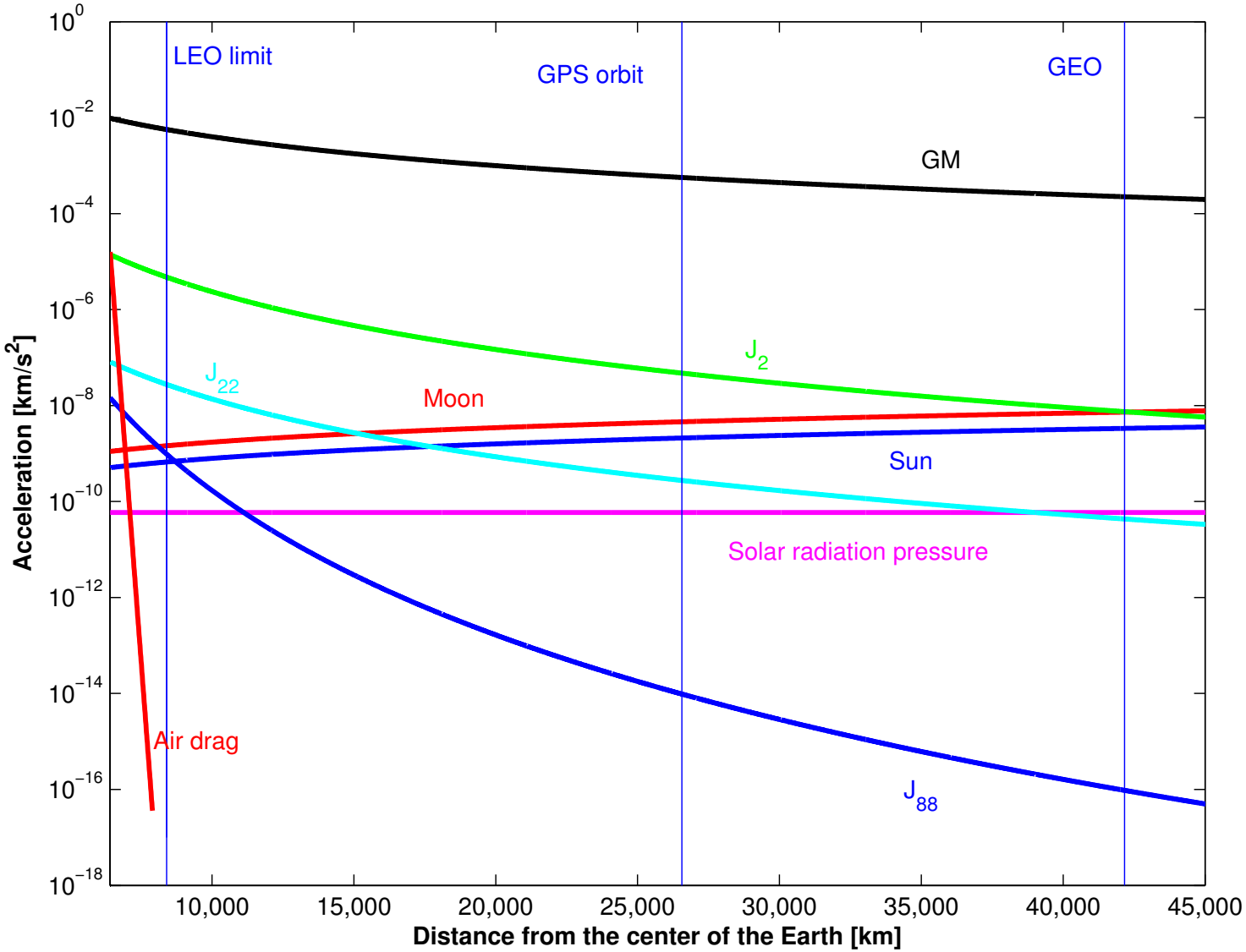
Orbital distribution



Orbital perturbations



Gravitational perturbations

Due to the non-spherical shape of the Earth.

$$V(r,\theta,\lambda) =$$

$$\frac{GM}{r}\left[1+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n P_{nm}(\sin\theta)(C_{nm}\cos(m\lambda)+S_{nm}\sin(m\lambda))\right]$$

r, θ, λ: radius, latitude and longitude (in a coordinate system whose origin is at the center of mass of the body)
G = universal constant of gravitation
M = mass of the body
R_e = characteristic physical dimension (e.g. the larger semi-axis of the body)
P_{nm} = associated Legendre functions
C_{nm}, S_{nm} = coefficients of the potential

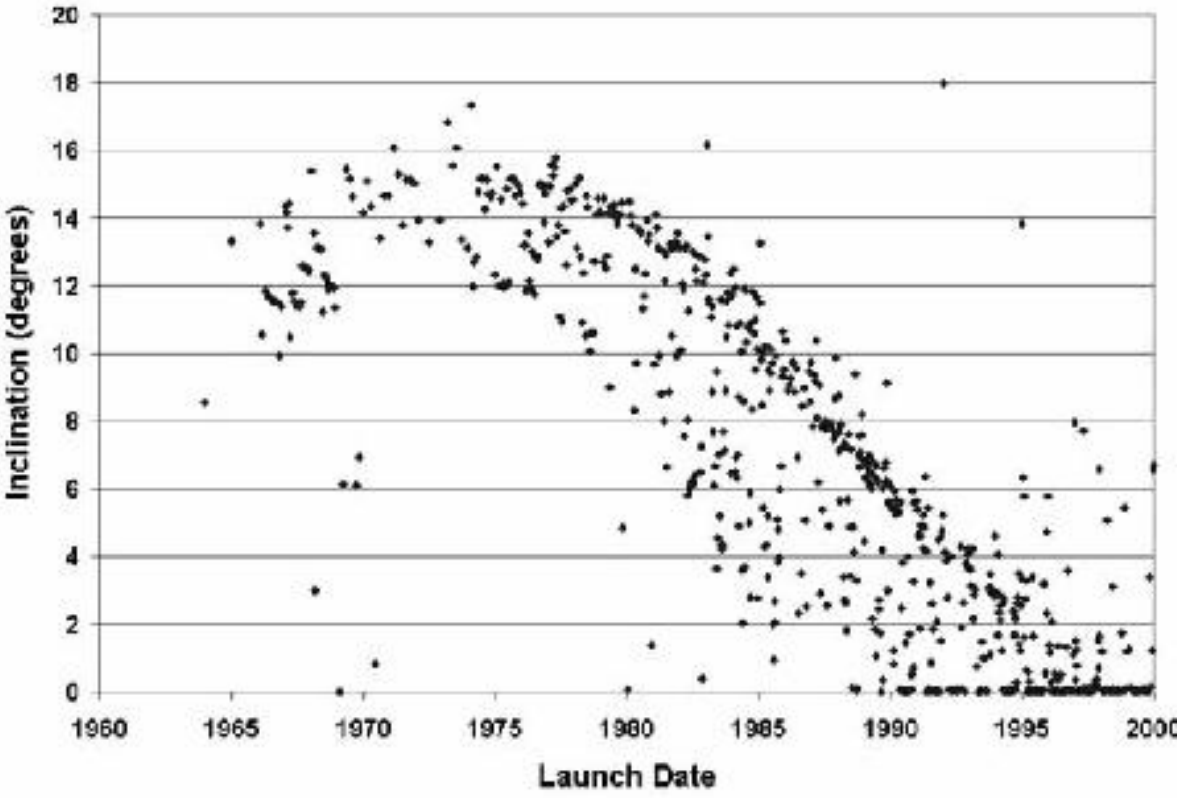
Gravitational perturbations

- Due to the non-spherical shape of the Earth.
- A satellite orbiting the Earth at an altitude h above the Earth's surface is affected by a constituent of the potential with harmonic degree ℓ less strongly than it would be at the surface.
- The amplitude being reduced by a factor $\left[\frac{a}{(a+h)}\right]^{(\ell+1)}$.
- Most of the effect is related to J_2, the quadrupole term of the gravity potential expansion in terms of spherical harmonics due to the Earth oblateness.
- Are important mainly in changing the angular arguments of the satellite orbit.
- The main effects of the geopotential perturbations are the secular regression of the orbital node, $\dot{\Omega}$ and the precession of the perigee argument ($\dot{\omega}$).

LUNISOLAR PERTURBATION

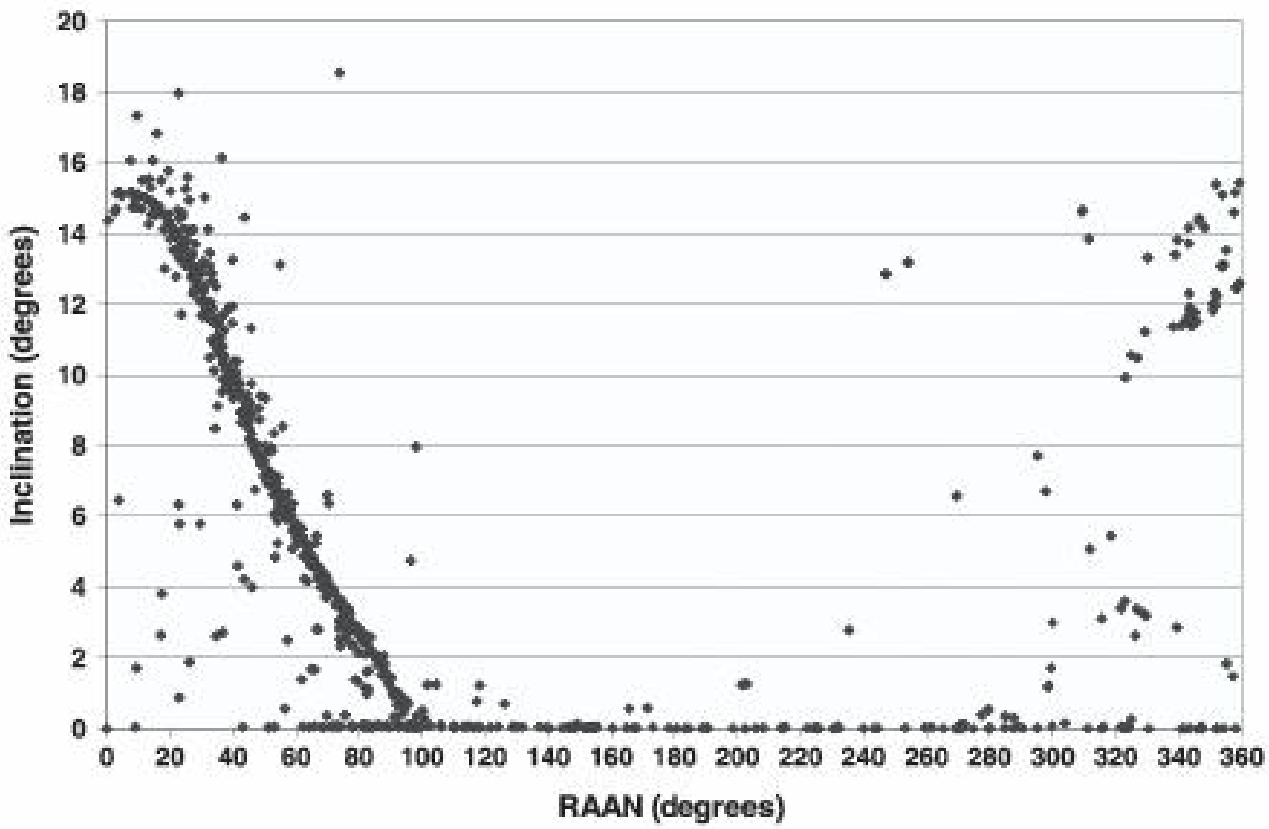
Attraction of the Sun and the Moon on the spacecraft ($\sim \frac{r}{r_{☾}^3}$)

- For orbits with periods equal to 12 h or longer, the lunisolar effects are significant and should be included.
- Long term (and/or secular) effects on e, i, Ω and ω.
- Oscillation ($P \sim 50$ y) around the stable Laplace plane at $i = 7.5°$ due to lunisolar perturbations.



LUNISOLAR PERTURBATION

- Systematic orientation of the orbital planes.
- Precession about pole of the ecliptic due to solar attraction + precession about pole of the Moon's orbit due to lunar attraction
- Correlation between an object's inclination and RAAN.



RESONANCES

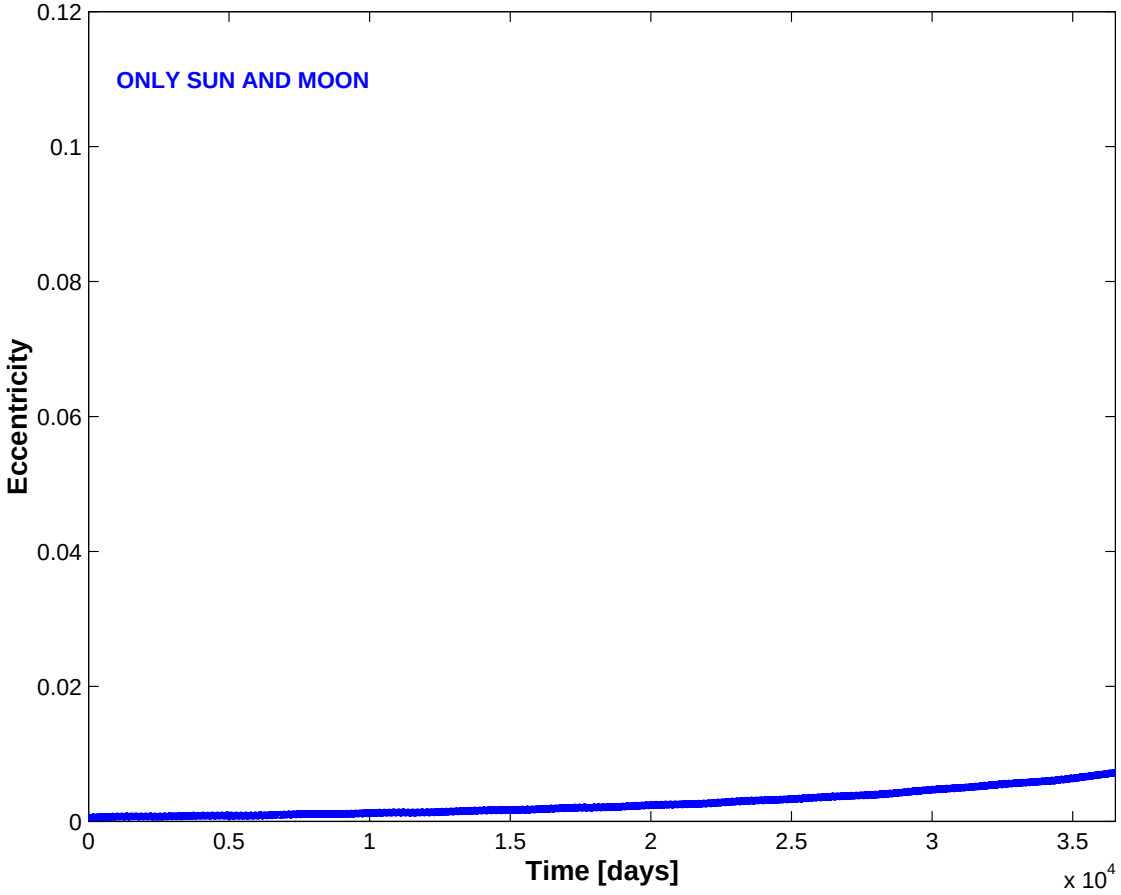
- Different kinds of resonances between the motion of the satellite, the rotation of the Earth and the motion of the perturbing bodies can produce long term and even secular perturbation exceeding those due to the simple action of a given perturbation alone.
- The navigation spacecraft in MEO have orbits whose period equals half a sidereal day (i.e., $\approx$ 12 hours) and are subject to the 2:1 mean motion resonance.
- In mean motion resonances, the exact condition for commensurability is that the satellite performs β nodal periods while the Earth rotates α times relative to the precessing satellite orbit plane (α and β are mutually prime integers, and for MEO take the values $\alpha = 1$ and $\beta = 2$). After this interval the path of the satellite relative to the Earth repeats exactly leading to the resonance.

RESONANCES

- The 2:1 resonance causes long period changes in the orbital eccentricity of MEO satellites and, for example, can modify the configuration of the navigation constellations.
- Recently a more complex resonance, resulting from the third body and the geopotential perturbations, was found to be the cause of a very long term-term (nearly secular) perturbation of the eccentricity of the navigation satellites orbits, representing a serious hazard for the long term disposal of the spent satellites and upper stages in the region (e.g., Rossi, CMDA, 2008).
- This luni-solar resonance appears when the secular motions of the lines of apsides and nodes become commensurable with the mean motion of the Sun and the Moon

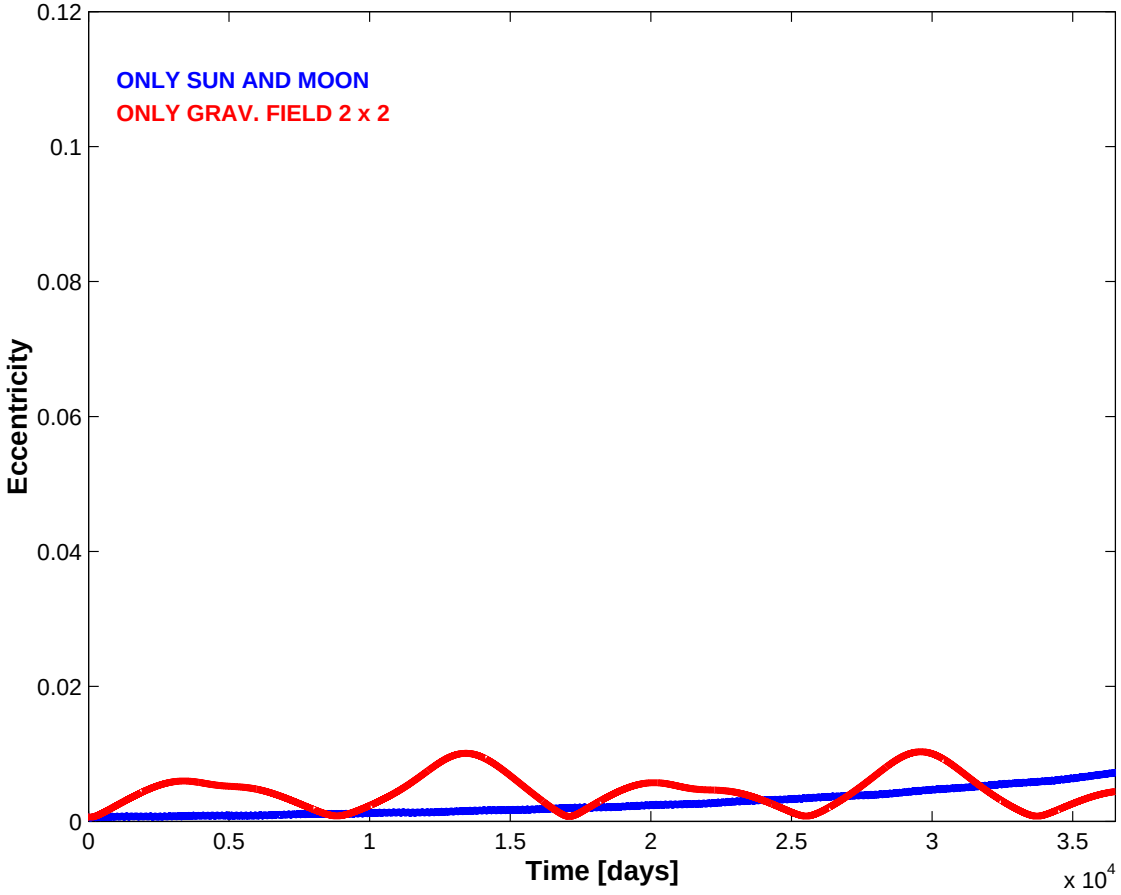
RESONANCE ONSET

Only Luni-Solar perturbations (no gravity harmonics)



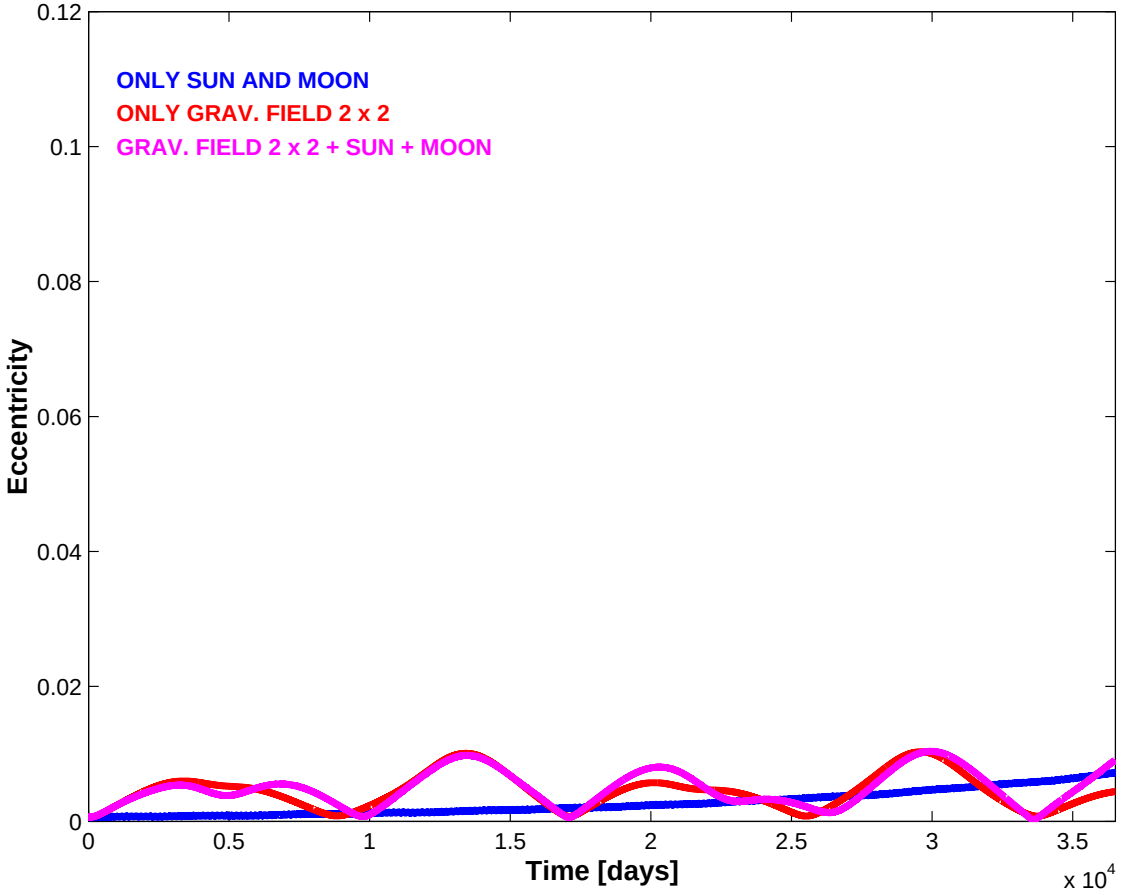
RESONANCE ONSET

Only gravity harmonics up to degree $\ell = 2$ and order $m = 2$.



RESONANCE ONSET

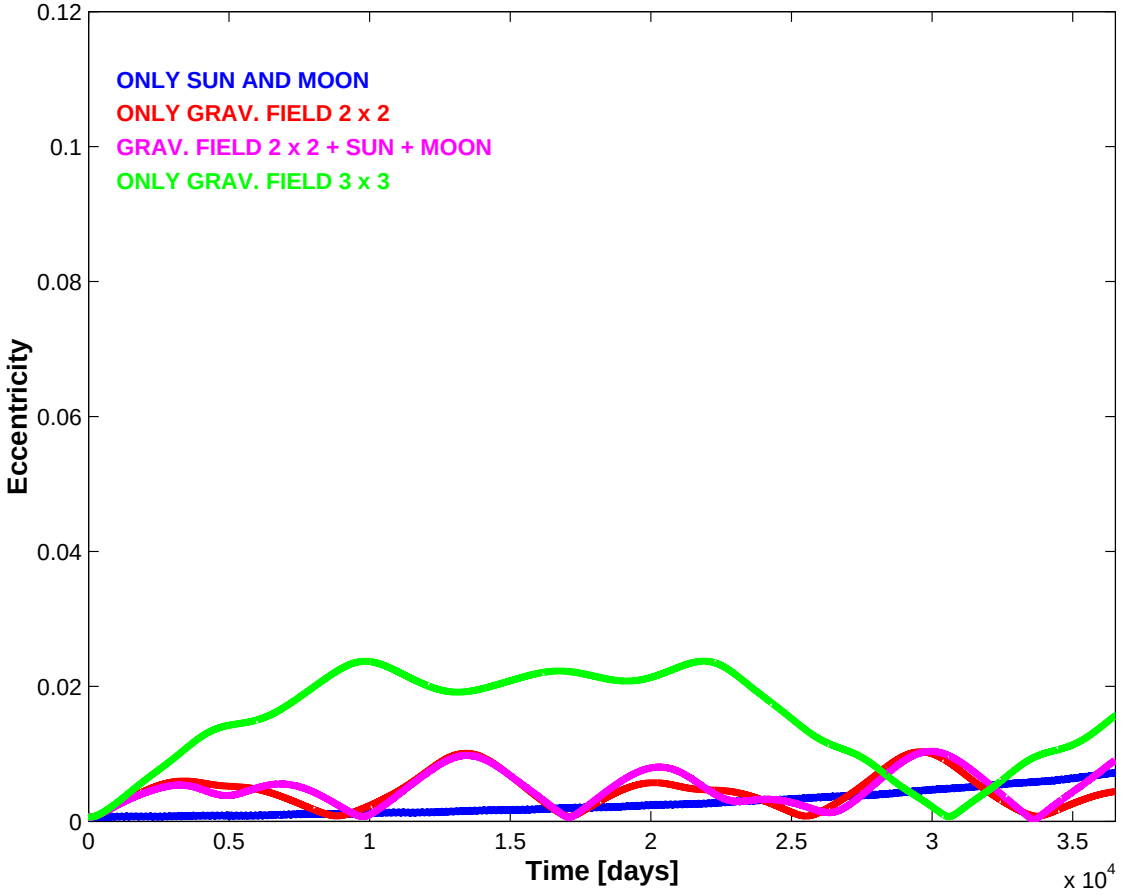
Gravity harmonics up to degree $\ell = 2$ and order $m = 2$ + Luni-Solar perturbations



RESONANCE ONSET

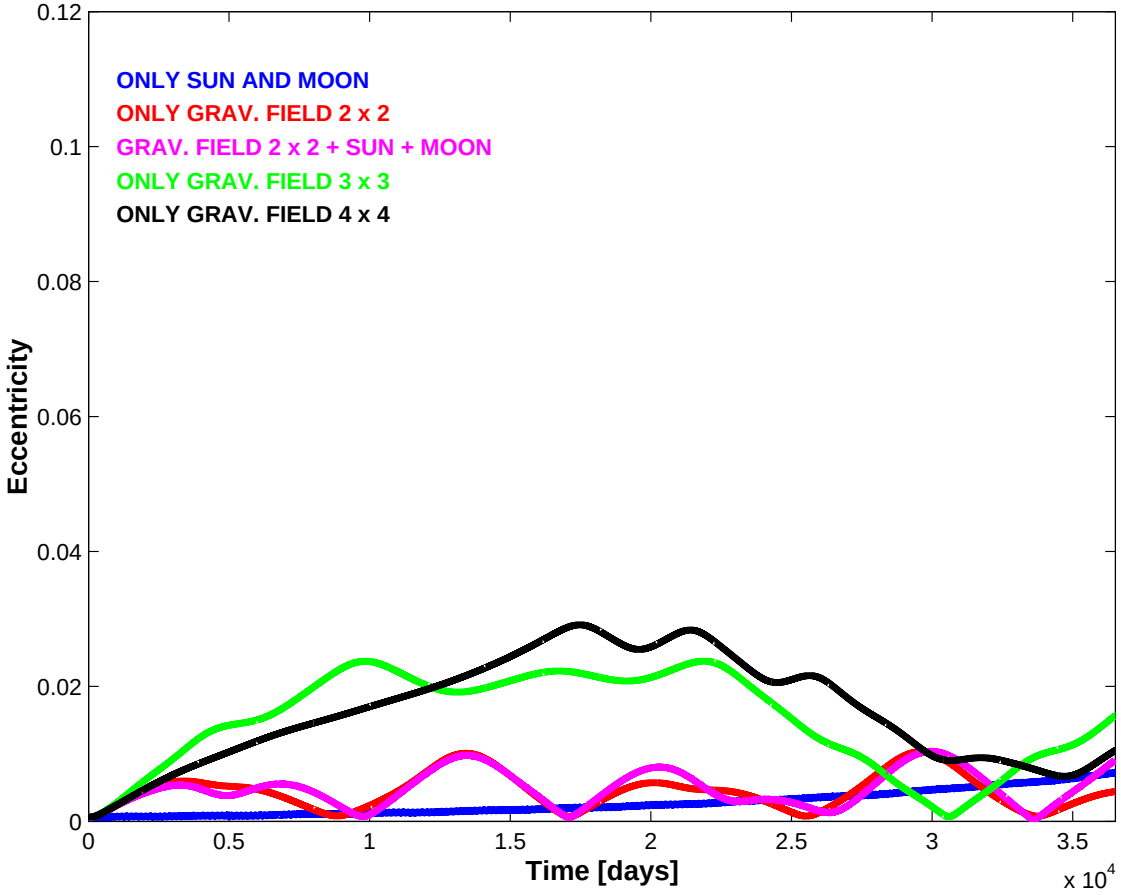
Only gravity harmonics up to degree $\ell = 3$ and order $m = 3$ (No Luni-Solar perturbations).

Long period perturbations from geopot. res.



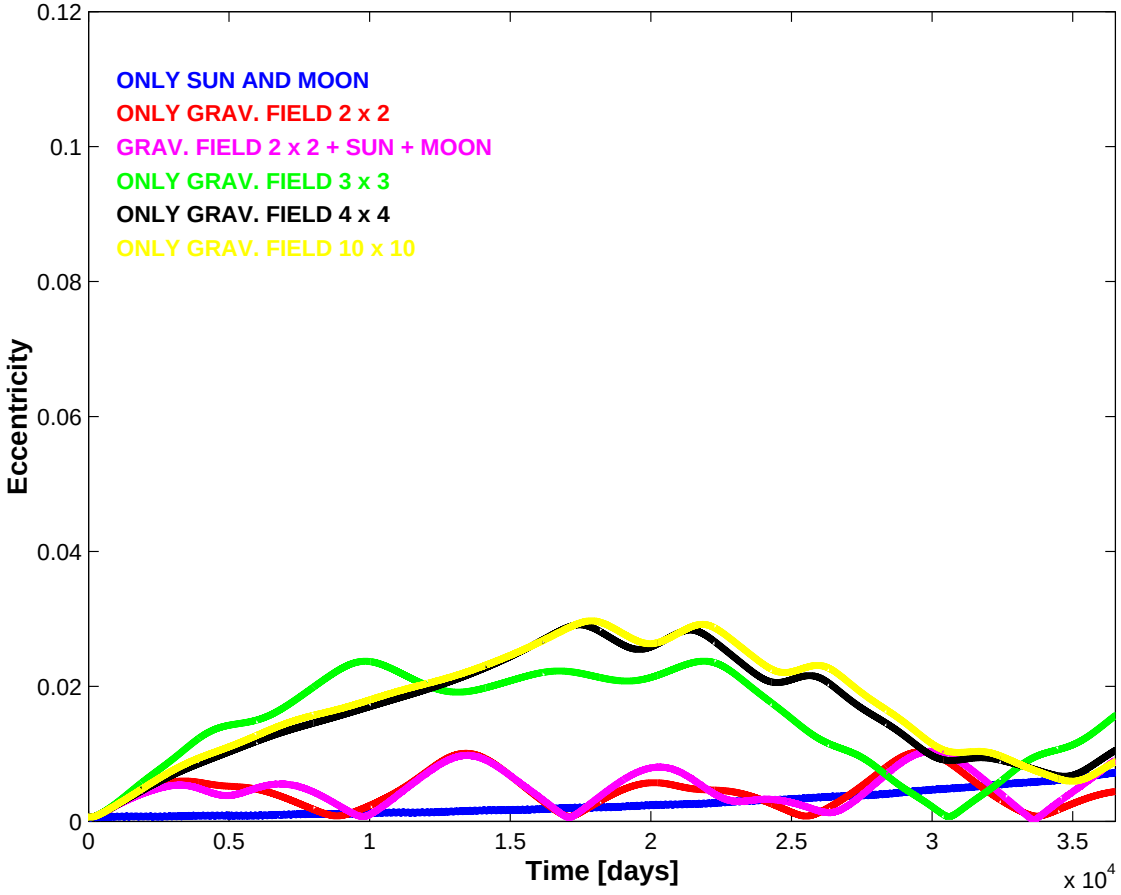
RESONANCE ONSET

Only gravity harmonics up to degree $\ell = 4$ and order $m = 4$ (No Luni-Solar perturbations).



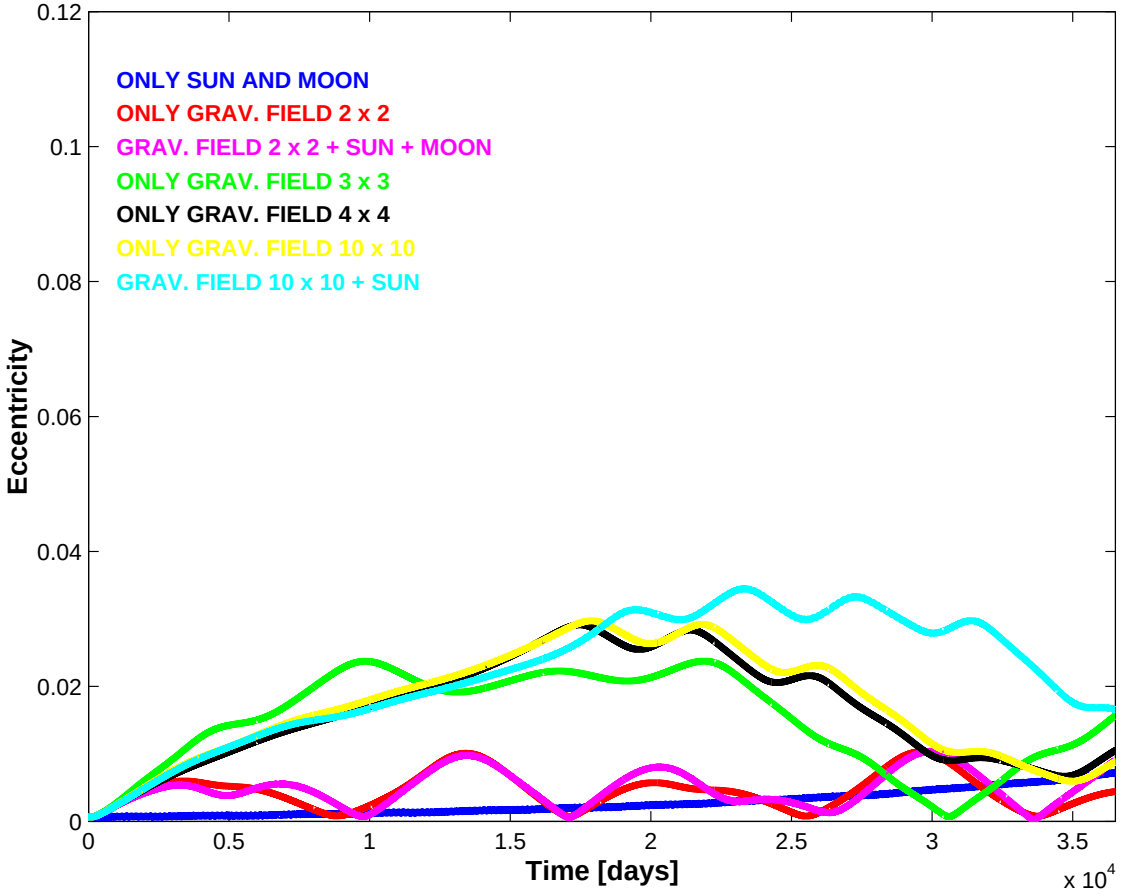
Resonance onset

Only gravity harmonics up to degree $\ell = 10$ and order $m = 10$ (No Luni-Solar perturbations).



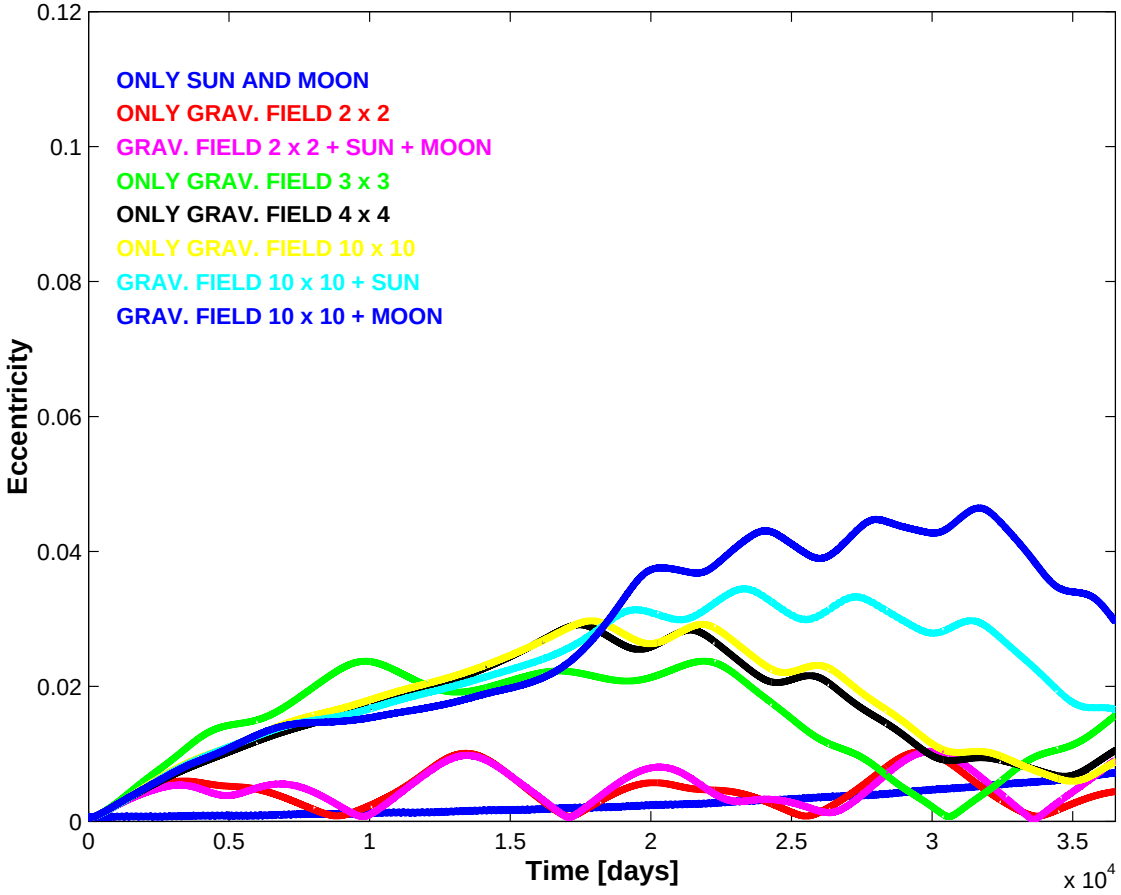
RESONANCE ONSET

Gravity harmonics up to degree $\ell = 10$ and order $m = 10$ + Solar perturbations (i.e., no Moon).



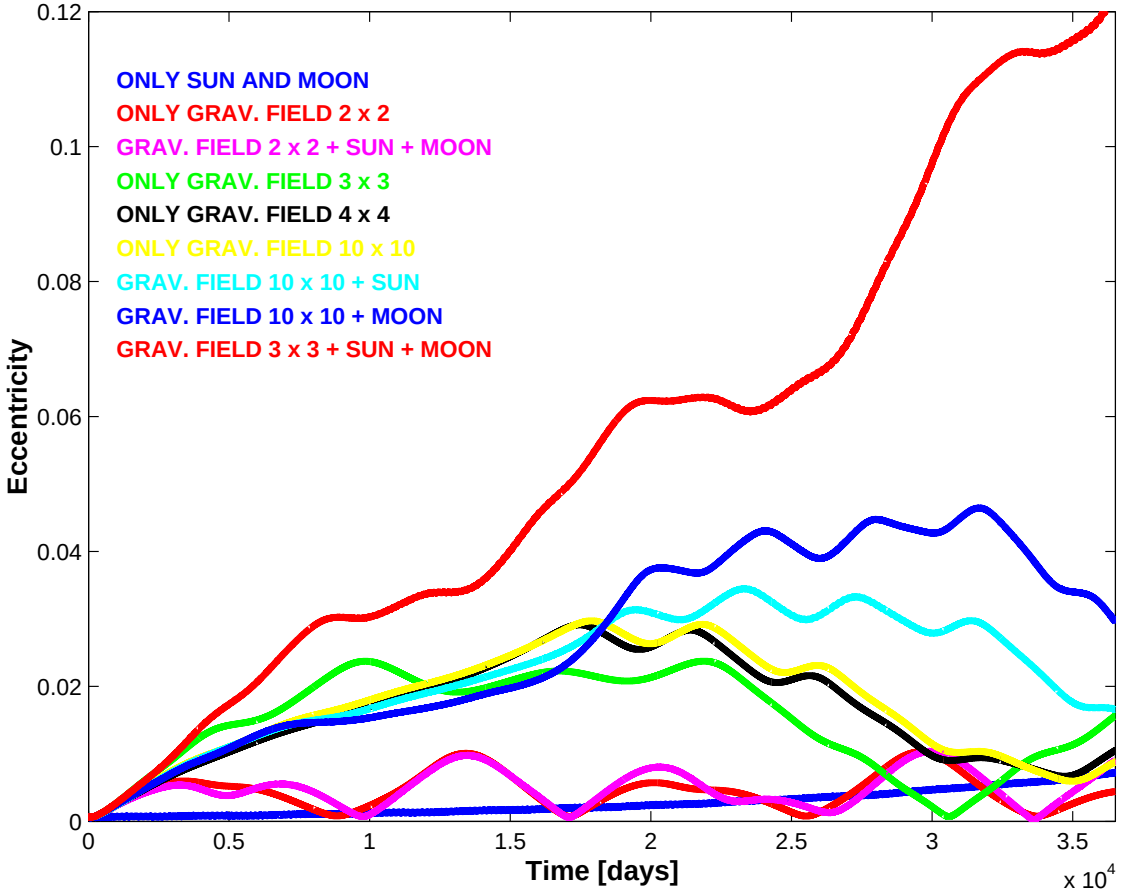
RESONANCE ONSET

Gravity harmonics up to degree $\ell = 10$ and order $m = 10$ + Lunar perturbations (i.e., no Sun).

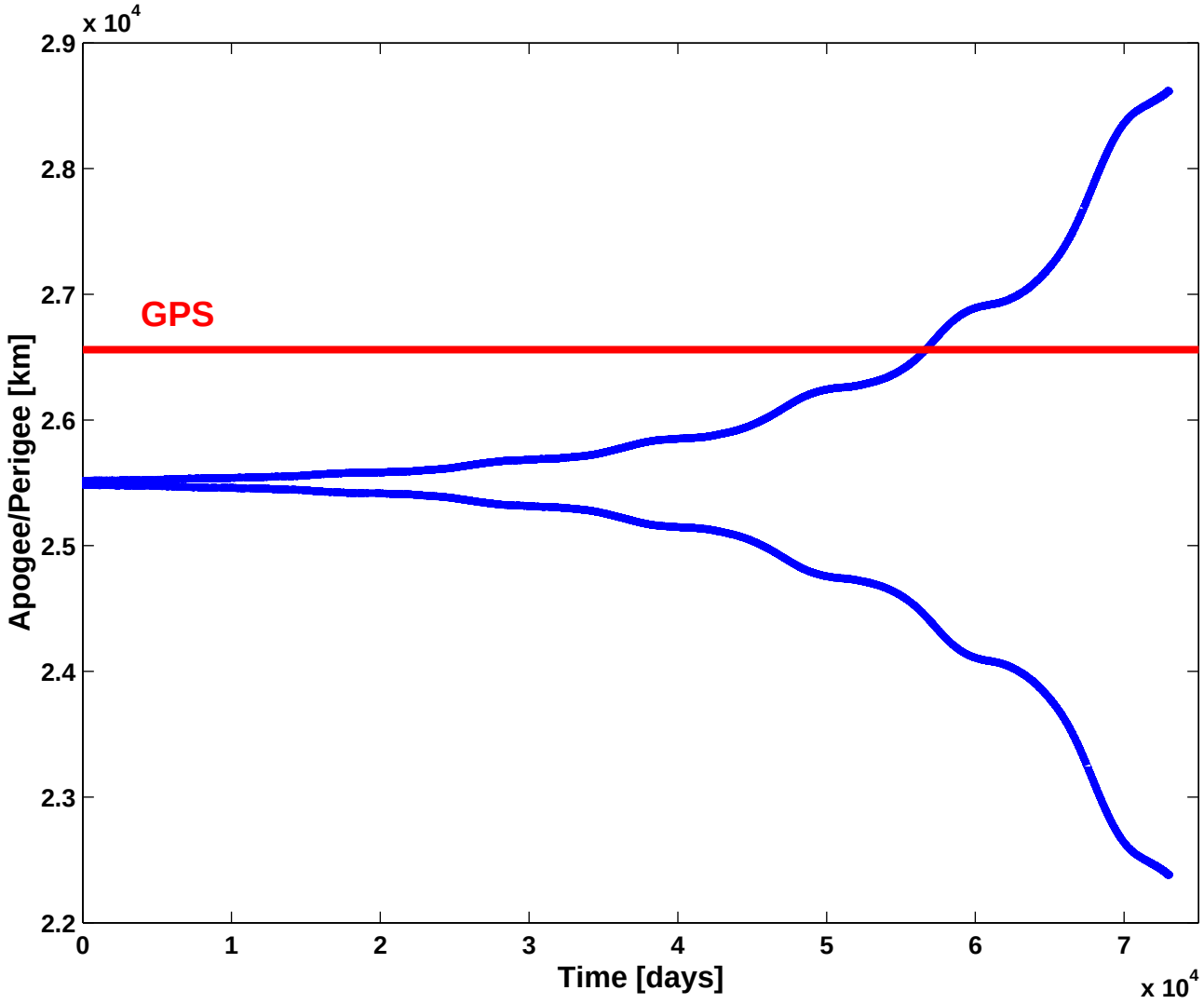


RESONANCE ONSET

Gravity harmonics up to degree $\ell = 3$ and order $m = 3$ + Lunar and Solar perturbations

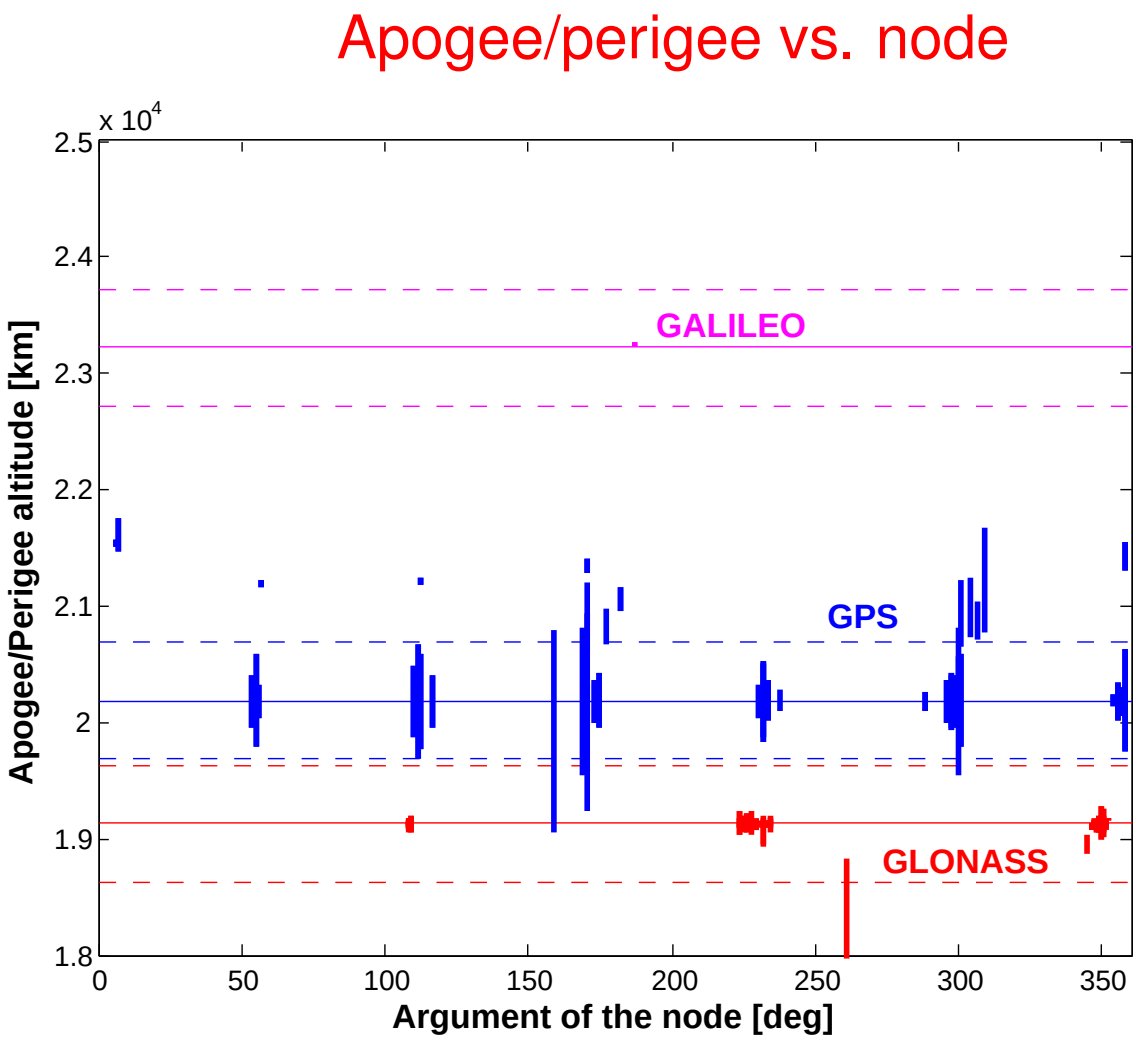


ETALON 2 - ORBIT EVOLUTION (200 Y)



SPACECRAFT + DEBRIS AROUND GNSSS

- 44 are GPS related objects
- 115 are GLONASS related objects



Non gravitational: Air drag acceleration

The air drag a_D is a non-conservative acceleration causing a secular decrease of the semimajor axis (i.e. a decay of the spacecraft into the atmosphere:

$$a_D = -\frac{1}{2} C_D \frac{A}{M} \rho v_r^2$$

A: cross sectional area of the spacecraft
$\rho(h)$: air density
C_D: dimensionless coefficient (~ 2) describing the interaction of the atmosphere with the satellite's surface materials
v_r: object's velocity with respect to the atmosphere
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$\rho(h)$: air density $\Longrightarrow \rho = \rho(h, t)$
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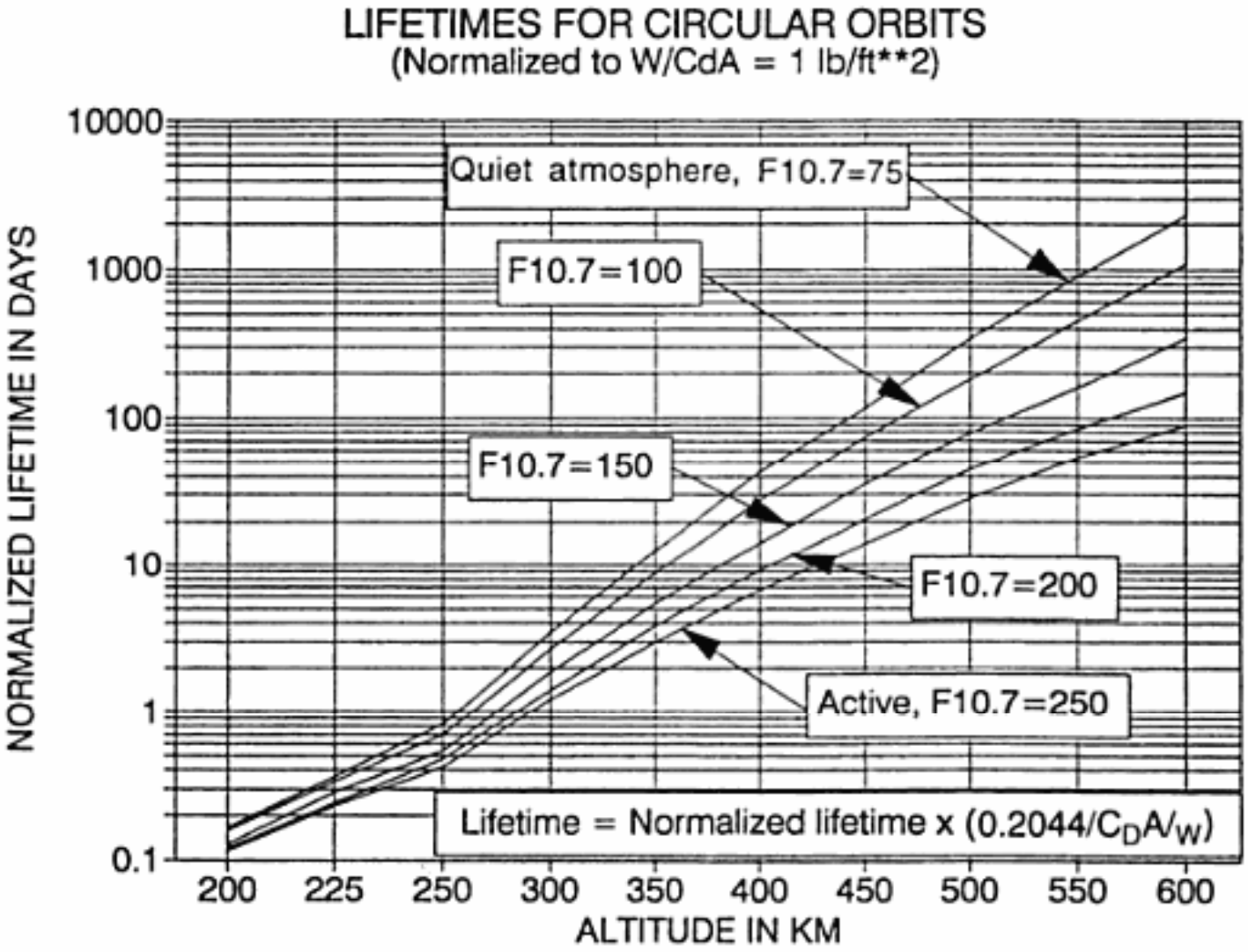
v_r: object's velocity with respect to the atmosphere
M: mass of the spacecraft

Air drag acceleration - GOCE

Air drag acceleration: lifetime



From: Chobotov, Orbital mechanics

Non gravitational: Solar radiation pressure

A satellite exposed to solar radiation experiences a small force that arises from the absorption or reflection of photons

- GEO sats. typically have large solar panels and antennas.
- Direct solar radiation pressure may significantly affect the eccentricity with small effects on the total energy of the orbit and, therefore, on the semi-major axis or mean motion.
- Solar radiation pressure acceleration:

$$\ddot{r} = -\frac{A}{M}\frac{\Phi_\odot}{c}C_R\frac{r_\odot}{r_\odot}^3$$

  - A: satellite cross sectional area
  - M: satellite mass
  - C_R: radiation pressure coefficient
  - $\Phi_\odot$: solar flux
  - c: velocity of light

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Typical A/M for a GEO satellite ~ 0.1 m^2 kg^{-1}

High A/M objects

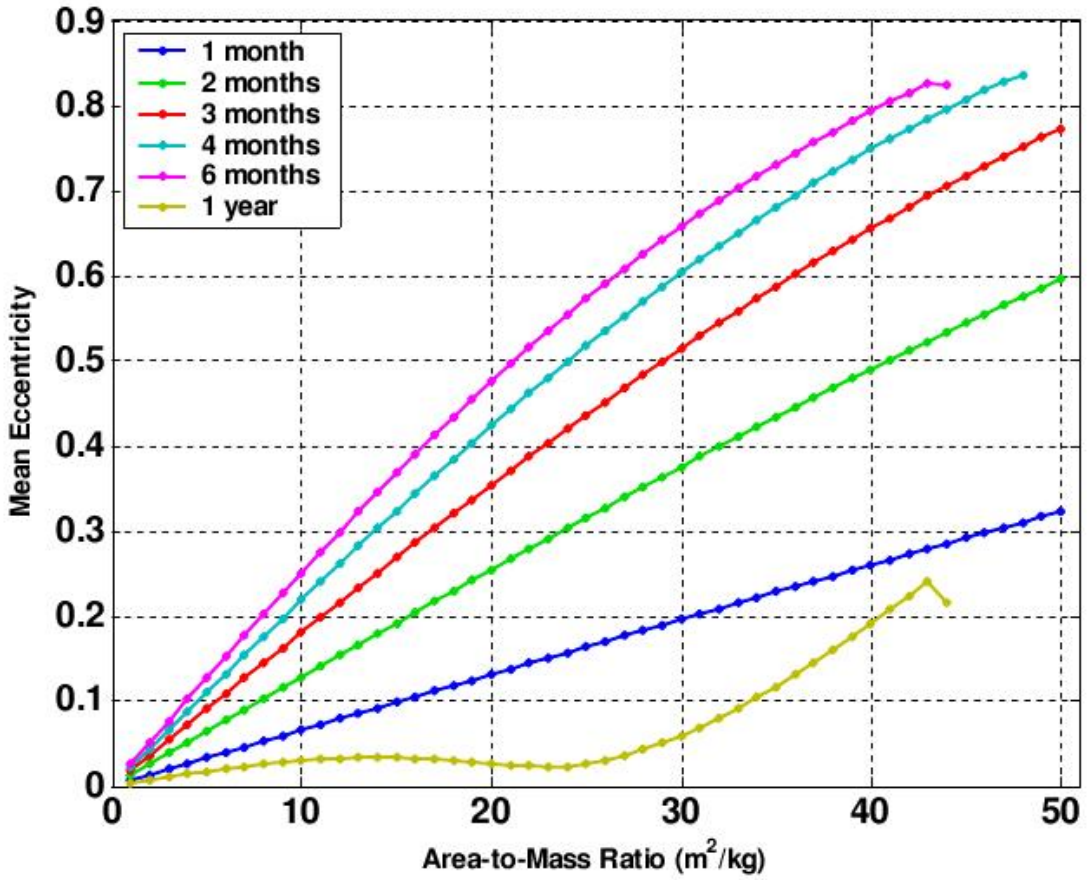
- Optical observations carried out by the ESA's 1-m telescope in Tenerife have identified a population of faint objects with mean motions of about 1 revolution per day and orbital eccentricities as high as 0.6
- The discovery of such objects was quite surprising....nothing is launched in or close to that orbit.
- Solar radiation pressure is the driving mechanism:

$$(a\dot{e})_{lp} \simeq \frac{1}{2\pi}\left(\frac{A\Phi_{\odot}}{Mc}\right)$$

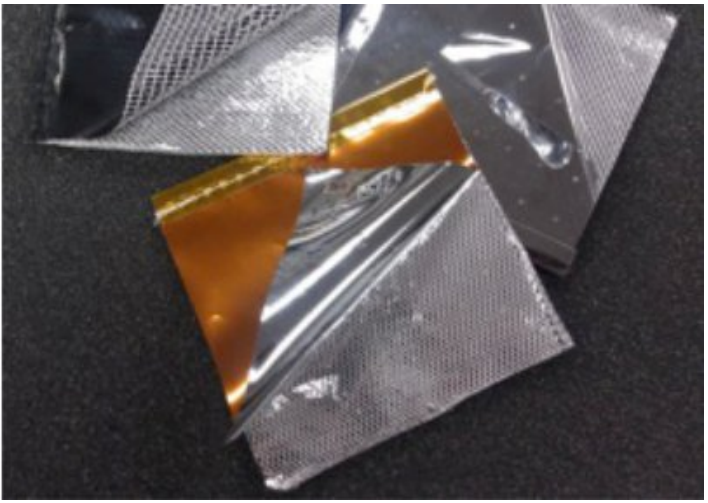
- However, this perturbation would be adequately effective only on objects with *extremely high area-to-mass ratios*.

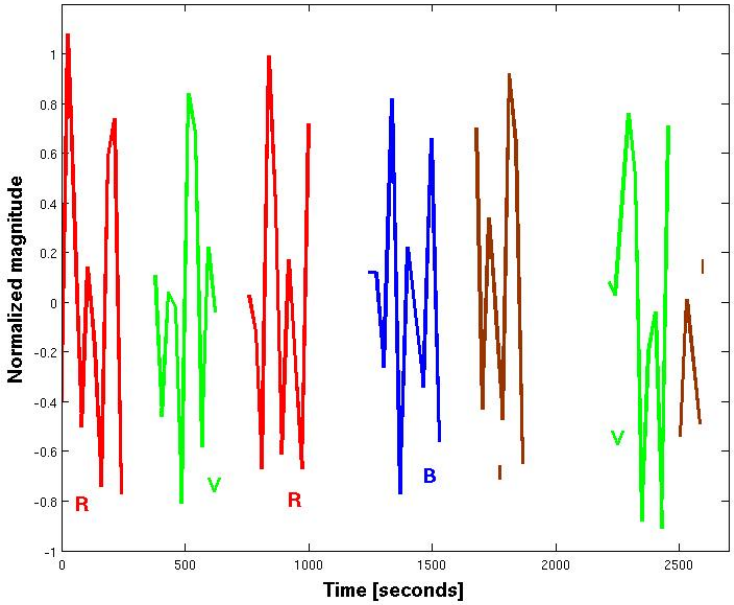
High A/M objects

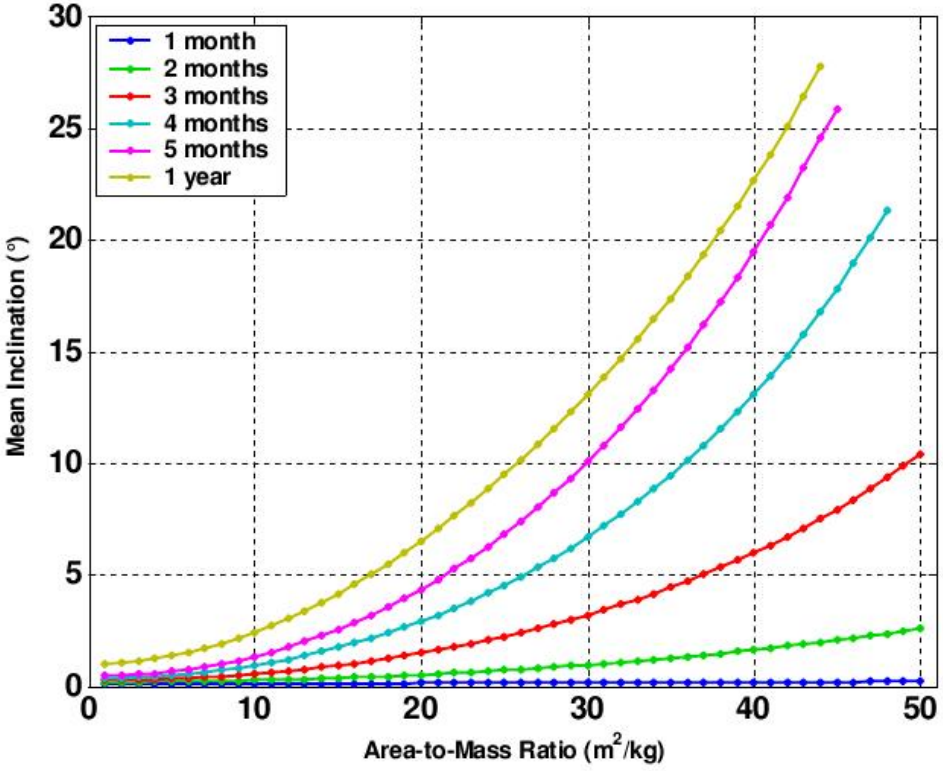
- To reach these orbits these objects must have $A/M \sim 30 \div 40$ m^2/kg
- Possible sources: thermal blankets or multi-layer insulation (MLI) (made from Mylar, Kapton or Nomex) either delaminated from aging spacecraft or ejected during explosions.



High A/M objects







Models of the future evolution

- In the late 80s a simple Volterra-like model was developed in Pisa by P. Farinella and A. Cordelli.
- From the early 90's mathematical models with increasing level of complexity were developed to study the evolution of the space debris population by our group in Pisa for the European Space Agency.
- Similar models were developed in the same years by NASA and, later on, by BNSC.

A SIMPLE VOLTERRA-LIKE MODEL

- The two population are the debris (projectiles), n and the satellites (targets).
- Generally speaking: *small vs large objects*.
- The differential equations are:

$$\begin{cases} \frac{dN}{dt} = A - xnN \\ \frac{dn}{dt} = \beta A + \alpha xnN \end{cases}$$

- Solving the system of equations, with different values of the constants, the combined evolution of the two population can be quantitatively described.

A SIMPLE VOLTERRA-LIKE MODEL

$$\begin{cases} \frac{dN}{dt} = A - xnN \\ \frac{dn}{dt} = \beta A + \alpha xnN \end{cases}$$

- $A =$ satellites growth rate: launched objects - objects re-entered in the atmosphere. This constant can be put to around 100.
- $x \sim 3 \times 10^{-10}$: constant such that xnN represents the number of collisions between projectiles and targets. Therefore xnN also represents the number of satellites destroyed by collisions. The value of x can be computed by a *particle-in-a-box* computation:
  - the number of collisions in the unit of time between two particles is $\sim V/W$, where V is the relative velocity and W is the volume;
  - In our case: $V \simeq 10$ km/s and $W \simeq 10^{12}$ km^3, so $V/W \simeq 10^{-11}$ km^{-2} s^{-1} $\simeq 3 \times 10^{-10}$ m^{-2} years^{-1}

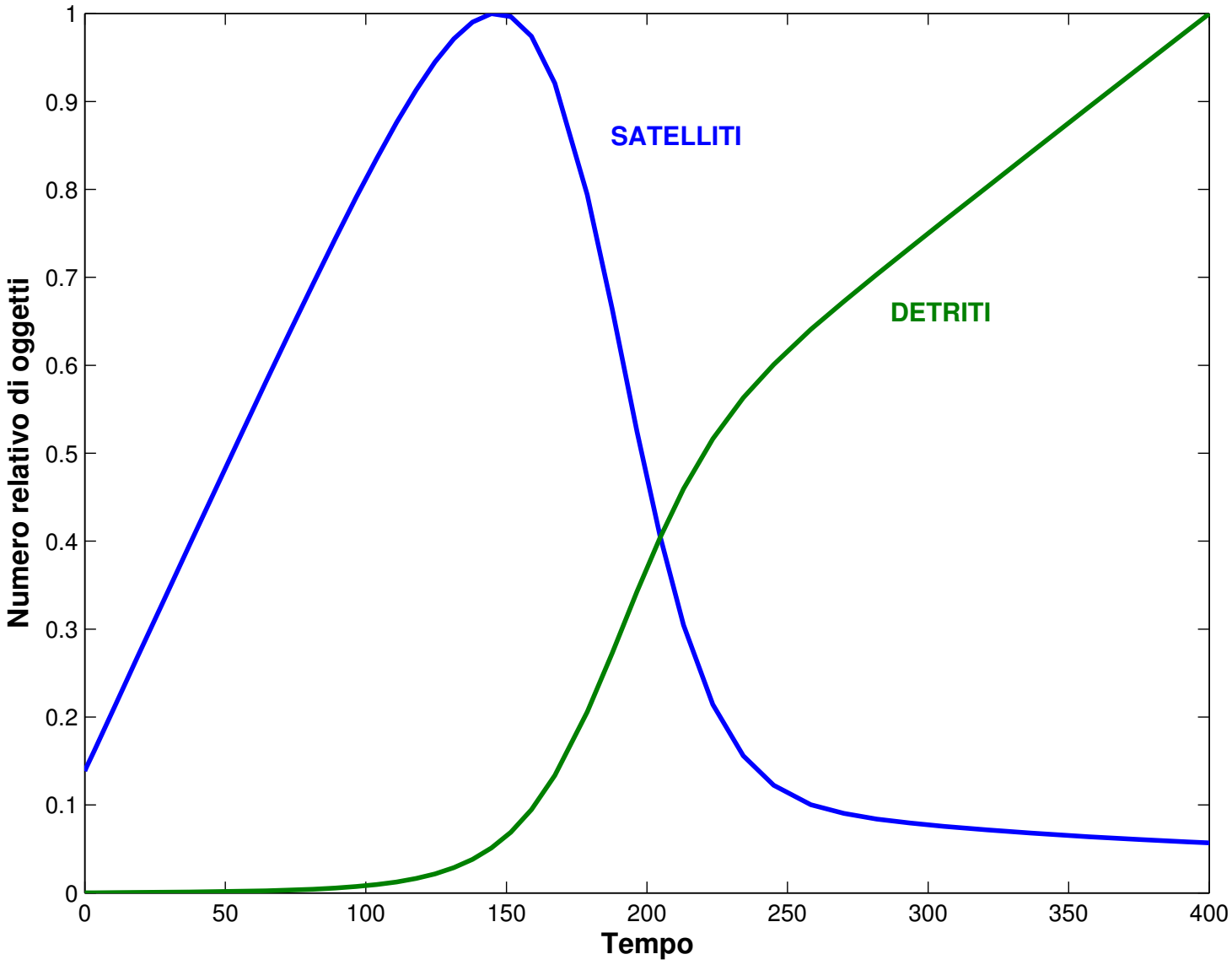
A SIMPLE VOLTERRA-LIKE MODEL

$$\begin{cases} \frac{dN}{dt} = A - xnN \\ \frac{dn}{dt} = \beta A + \alpha xnN \end{cases}$$

where:

- $\beta \simeq 70$: mean number of *primary fragments*, i.e. generated by explosions or mission related objects.
- $\alpha \simeq 10^4$: number of fragments produced in a typical collision. Therefore αxnN gives the number of objects produced in xnN collisions.
- To solve the system we just need suitable initial conditions, e.g. in the paper the authors used $N_0 = N(0) = 2 \times 10^3$ and $n_0 = n(0) = 5 \times 10^4$.

A simple Volterra-like model



Towards increased complexity.....

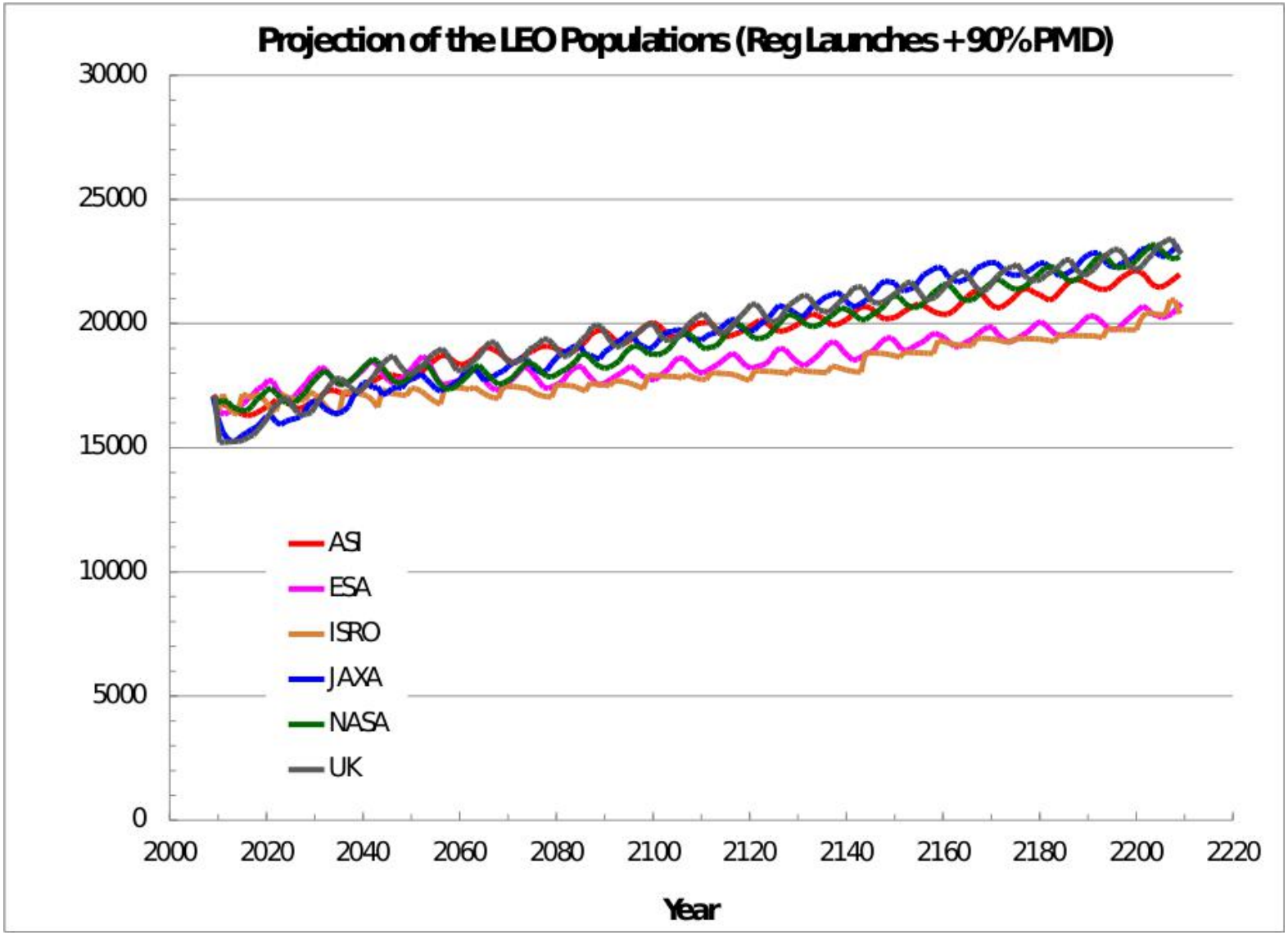
In Rossi, Cordelli and Farinella (*JGR, 1994*) the model was extended to multiple altitude shells and bins of mass for the objects:

$$\frac{dN(m_i, h_j, t)}{dt} = \beta(m_i, h_j) - \frac{N(m_i, h_j)}{\tau(m_i, h_j)} + \frac{N(m_i, h_{j+1})}{\tau(m_i, h_{j+1})} + \sum_{k,l} f(m_k, m_l, m_i) p(h_j) \sigma(m_k, m_l) N(m_k, h_j) N(m_l, h_j) \quad (4)$$

THE LAST MODELS (SDM)

- The need for detailed studies of measures to mitigate the growth of the space debris called for more detailed simulation models dealing with single objects.
- The orbit of all the (larger) objects are propagated individually with a semi-analytic propagator including geopotential harmonics, third body perturbations, solar radiation pressure (including shadows) and atmospheric drag.
- All the source and sinks processes are modelled separately:
  - Launches
  - Explosions
  - Collisions
  - RORSATs like events
  - Solid rocket motors exhausts (slag).
  - Mitigation measures, collision avoidance and active debris removal.

SDM: NUMBER OF OBJECTS LARGER THAN 10 CM



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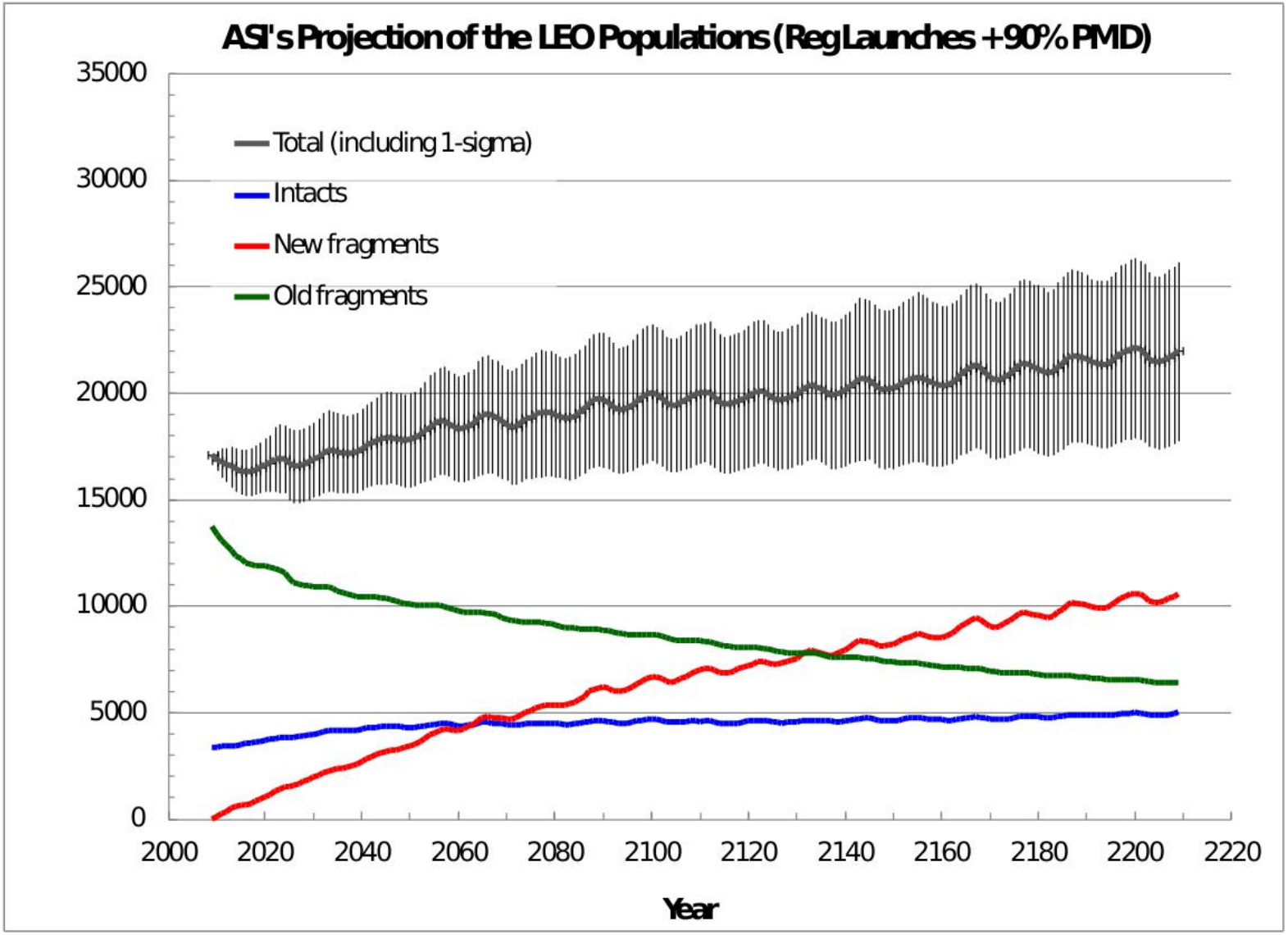

Future modelling work

- Asses the need of active debris removal activities.
- Identify the best candidates for active debris removal.
- Asses the level of active debris removal activities.
- Study effective disposal measures for high Earth orbits (GNSSs, etc....)
- Map the stability/instability of the Earth orbital regions.
- Share and spread the awareness of the debris problems in space.....

Future modelling work