

Stardust OTS – Rotational Dynamics and Attitude Control

Contents

Introduction

Basic motion

The Attitude Control Subsystem

Rotational dynamics

ACS for Rendezvous

Rendezvous

Conclusions

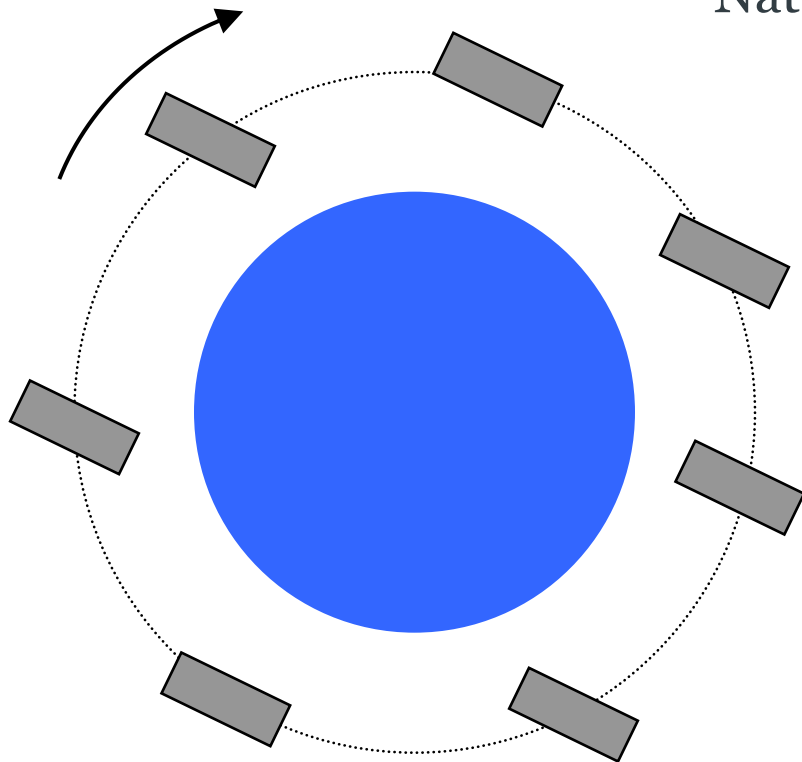
Introduction

Aim of lectures

- To discuss and reinforce the concepts of rotational dynamics and angular momentum in general
- Discuss these concepts when applied to an active debris removal mission
- To provide a greater understanding into the requirements of the attitude and orbit control systems of a chaser spacecraft and its effect on the satellite design.

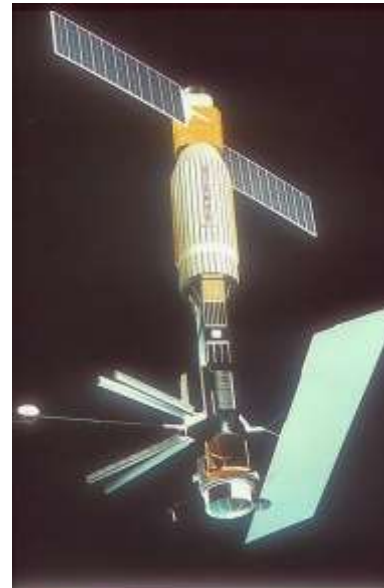
Basic motion

Consider a body in orbit (with no disturbances)...



Naturally occurring (disturbance) torques:

- Aerodynamic $< \sim 500$ km
- Magnetic $\sim 500 - 35,000$ km
- Solar radiation $> \sim 600 - 700$ km
- Gravity gradient $\sim 500 - 10,000$ km



Note that: the altitude ranges given are very approximate

‘SeaSat’ - Example of satellite affected by gravity gradients

The Attitude Control Subsystem (ACS*)

Prime purposes:

- To achieve the pointing requirements of the payload
 - directions and accuracye.g. Antennas - Earth pointing, say, or
Telescopes - diverse directions etc
- To achieve the pointing requirements for 'house-keeping'
 - in all phases of the missione.g. Power-raising - Sun-pointing
Communications - Earth-pointing
Thermal - Deep space
Orbit change thruster - as required
- To manage the (angular) momentum

*Note that: this subsystem is often referred to as the Attitude and Orbit Control Subsystem (AOCS)

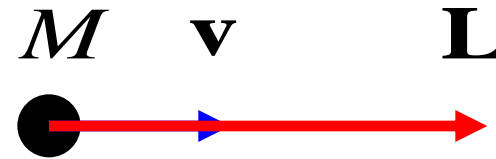
Rotational dynamics

Linear Momentum

- a 'stepping-stone' to translational/orbit dynamics

$$\mathbf{L} = M\mathbf{v}$$

\swarrow \swarrow \searrow
 vector scalar vector



Newton's second law:

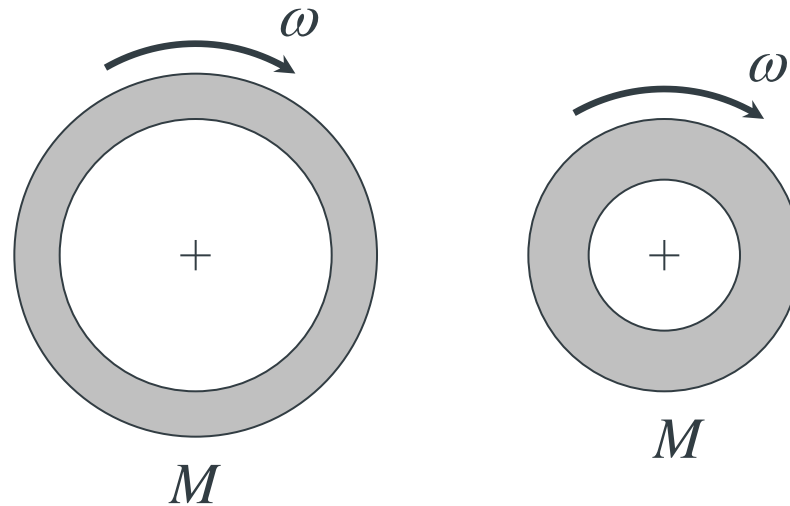
$$\frac{d}{dt}(\mathbf{L}) = \frac{d}{dt}(M\mathbf{v}) = \sum \mathbf{F}_{ext}$$

Free Motion:

$$\text{No Force, } \sum \mathbf{F}_{ext} = \mathbf{0} \Rightarrow \text{Momentum } \mathbf{L} \text{ is constant}$$

Rotational dynamics

Angular momentum – Inertia, one dimension



Is the angular momentum the same?

Rotational dynamics

Angular momentum – Rotational vectors

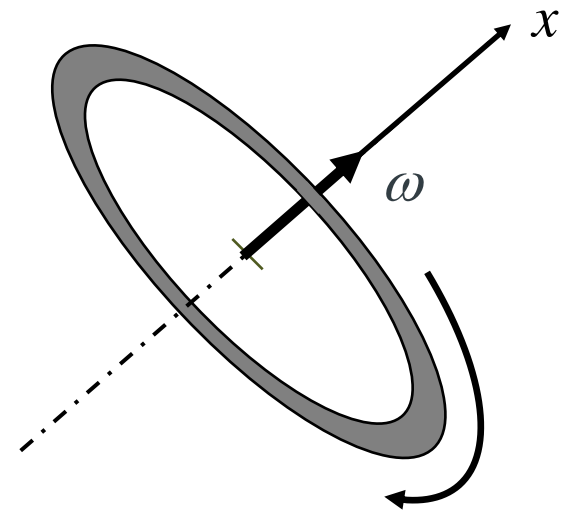
For this one dimensional motion:

$$\mathbf{H} = I_{xx} \boldsymbol{\omega}$$

\swarrow \swarrow \swarrow
 vector scalar vector

Newton's second law:

$$\frac{d}{dt}(\mathbf{H}) = \frac{d}{dt}(I_{xx} \boldsymbol{\omega}) = \sum \mathbf{T}_{ext}$$



Free Motion:

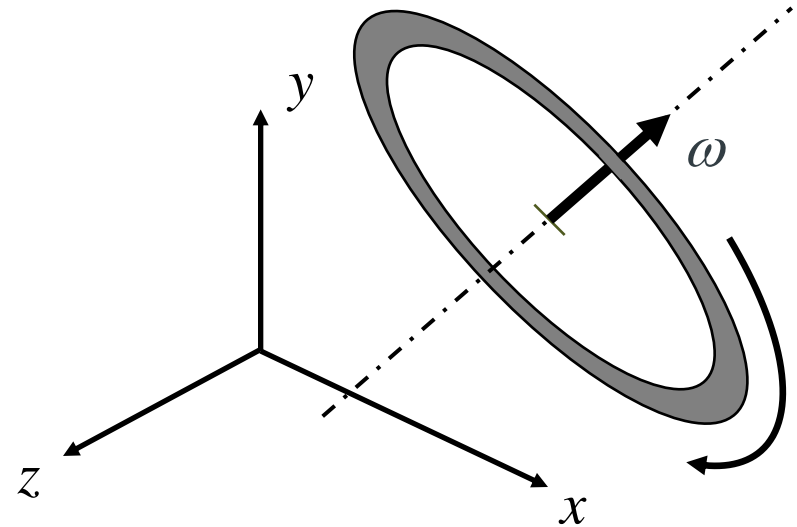
No torque, $\sum \mathbf{T}_{ext} = 0 \Rightarrow$ Momentum \mathbf{H} is constant

Rotational dynamics

Angular momentum

In three dimensions:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

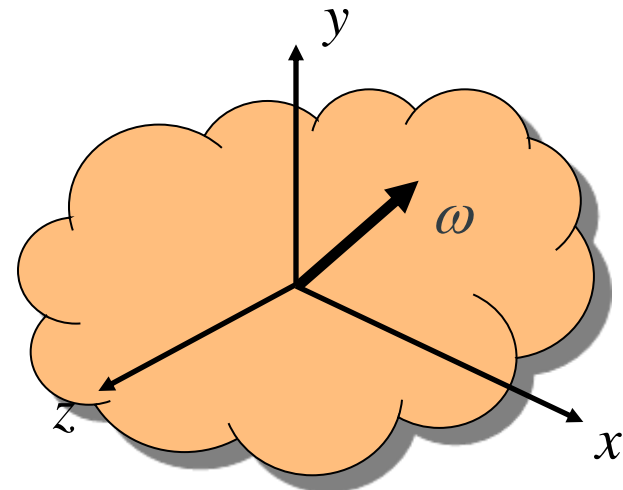


Angular momentum:

$$\mathbf{H} = \mathbf{I}\boldsymbol{\omega}$$

vector

matrix vector



Rotational dynamics

Angular momentum

Angular momentum of a rigid body such as the main structure of a spacecraft is:

$$\mathbf{H}_C = \mathbf{I}_C \boldsymbol{\omega}$$

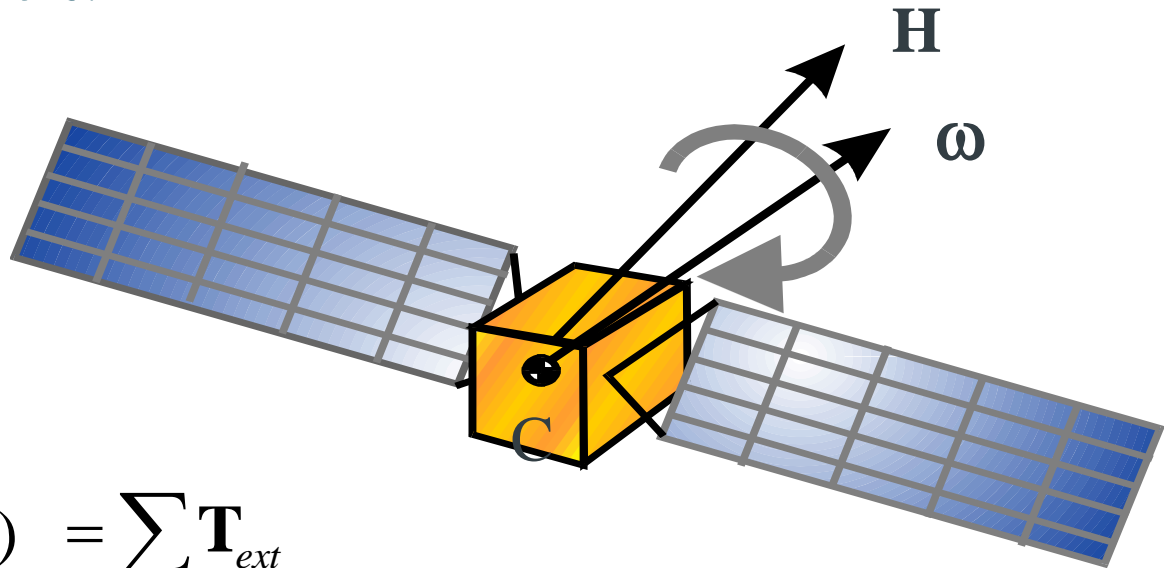
Inertia matrix referred to the centre of mass 'C'

Newton's second law:

$$\frac{d}{dt}(\mathbf{H}_C) = \frac{d}{dt}(\mathbf{I}_C \boldsymbol{\omega}) = \sum \mathbf{T}_{ext}$$

Free Motion:

$$\text{No torque, } \sum \mathbf{T}_{ext} = 0 \Rightarrow \text{Momentum } \mathbf{H} \text{ is constant}$$



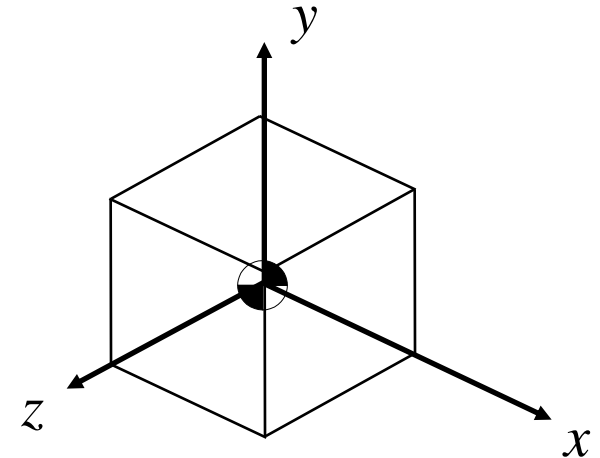
Rotational dynamics

The inertia matrix

The inertia matrix referred to the centre of mass:

$$\mathbf{H}_C = \mathbf{I}_C \boldsymbol{\omega}$$

$$\mathbf{I}_C = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$



I_{xx}, I_{yy}, I_{zz} are **Moments of Inertia**

I_{xy}, I_{yz}, I_{zx} are **Products of Inertia**

Products of inertia are a measure of unbalance, and cause ‘cross-coupling’

Rotational dynamics

Angular momentum components

$$\mathbf{H}_C = \mathbf{I}_C \boldsymbol{\omega}$$

$$\mathbf{H}_C = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

So the components of the angular momentum vector are:

$$\mathbf{H}_C = \begin{pmatrix} \left(I_{xx} \omega_x - I_{xy} \omega_y - I_{xz} \omega_z \right) \\ \left(I_{yy} \omega_y - I_{yz} \omega_z - I_{xy} \omega_x \right) \\ \left(I_{zz} \omega_z - I_{xz} \omega_x - I_{yz} \omega_y \right) \end{pmatrix}$$

Rotational dynamics

The inertia matrix

Moments of inertia:

$$I_{xx} = \int (y^2 + z^2) dm$$

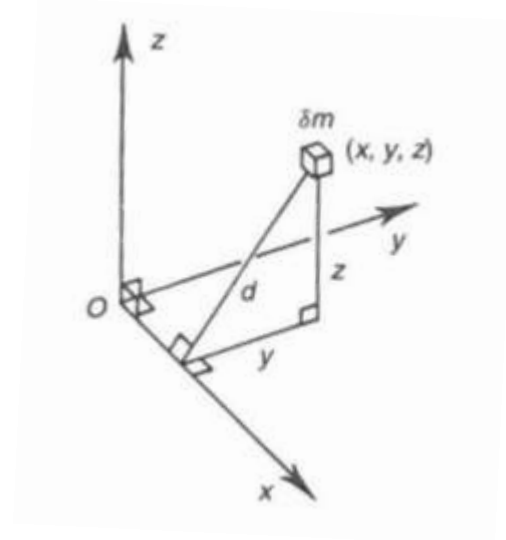
Where the integral extends over the whole mass distribution

Products of inertia:

The product of inertia associated with the x-axis is:

$$I_{yz} = \int yz dm$$

Generally these values are based on standard shapes with known formulae



Rotational dynamics

The inertia matrix

$$\mathbf{I}_C = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$$

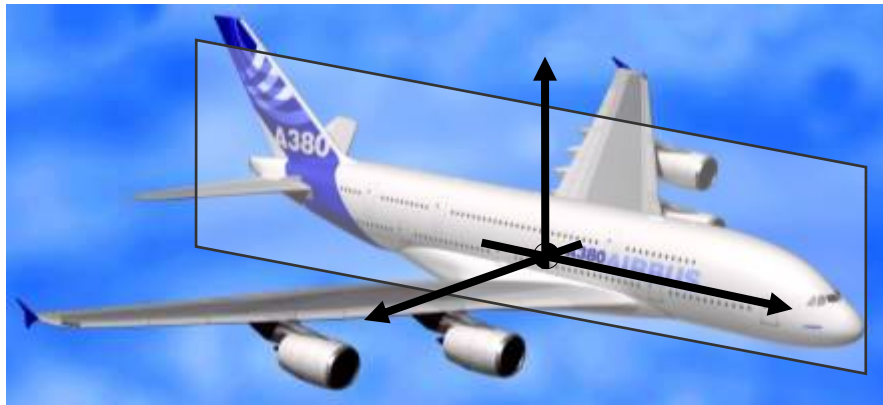
Therefore for a single particle of mass around point C:

$$\mathbf{I}_{Cm} = \begin{pmatrix} m(y^2 + z^2) & -mxy & -mxz \\ -mxy & m(x^2 + z^2) & -myz \\ -mxz & -myz & m(x^2 + y^2) \end{pmatrix}$$

Rotational dynamics

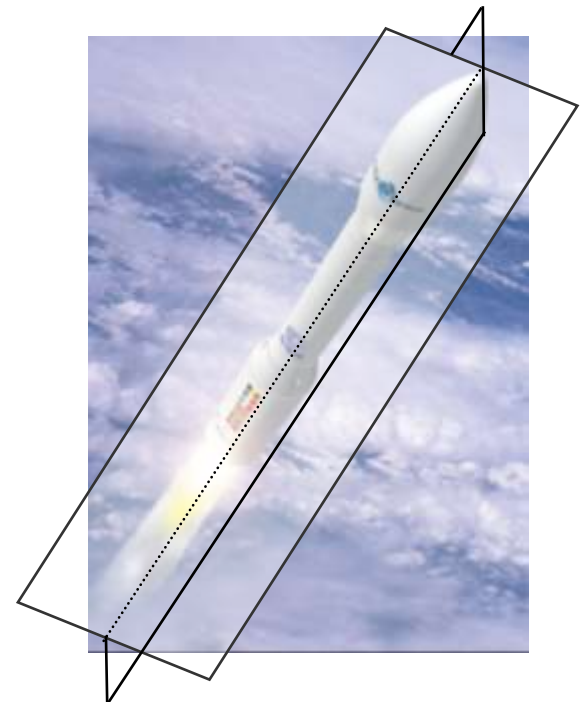
The inertia matrix – products of inertia

If there is a plane of symmetry, then the product of inertia associated with all axes in that plane will be zero. For example, an aircraft whose xz-plane is a plane of symmetry will have:



$$I_{xy} = 0 \quad I_{yz} = 0$$

If two of the co-ordinate planes are planes of symmetry, then all three of the products of inertia will be zero. This applies to axially symmetric bodies such as many expendable launchers.



Rotational dynamics

Useful Formulae

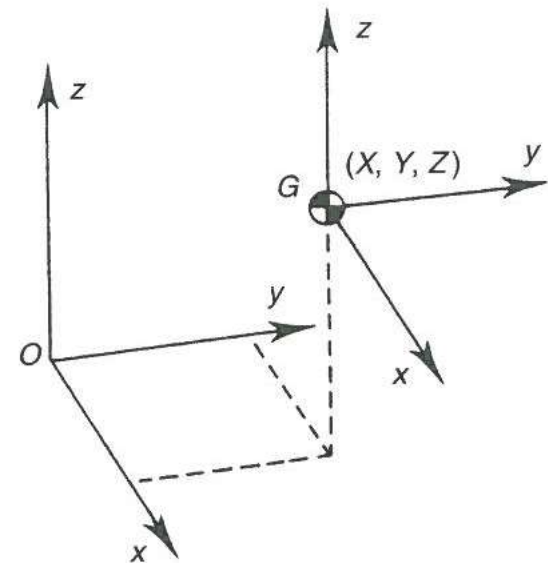
Transfer of reference point (parallel axis theorem)

If an object whose centre-of-mass G is at (X, Y, Z) has an inertia matrix $[I_G]$ referred to G , then add on the inertia matrix of its equivalent particle referred to O , in order to obtain the inertia matrix $[I_O]$ referred to parallel axes at O , that is:

$$[I_O] = [I_G] + [I_{OM}]$$

For a point mass/idealised component:

$$[I_{CM}] = \begin{pmatrix} M(y^2 + z^2) & -Mxy & -Mxz \\ -Mxy & M(x^2 + z^2) & -Myz \\ -Mxz & -Myz & M(x^2 + y^2) \end{pmatrix}$$



Rotational dynamics

Useful Formulae

Rotated axes theorem

If the components of a vector \mathbf{V} in one set of axes are expressed as the terms in a (3×1) column matrix \mathbf{V}_1 , say, and \mathbf{V}_2 consists of its components in a second set that rotated relative to the first, then \mathbf{V}_2 may be expressed as:

$$\mathbf{V}_2 = [\mathbf{R}]\mathbf{V}_1$$

Then $[\mathbf{R}]$ is known as a rotation matrix.

The inertia matrix $[\mathbf{I}]$ can then be transformed between the same set of axes by using:

$$[\mathbf{I}_2] = [\mathbf{R}][\mathbf{I}_1][\mathbf{R}]^{-1}$$

The rotation matrix can be constructed using Euler angles.

Rotational dynamics

Useful Formulae

Moment of inertia about a single arbitrary axis

If an inertia tensor is specified for the axes x , y and z , the moment of inertia of the body about an inclined axis can be computed using:

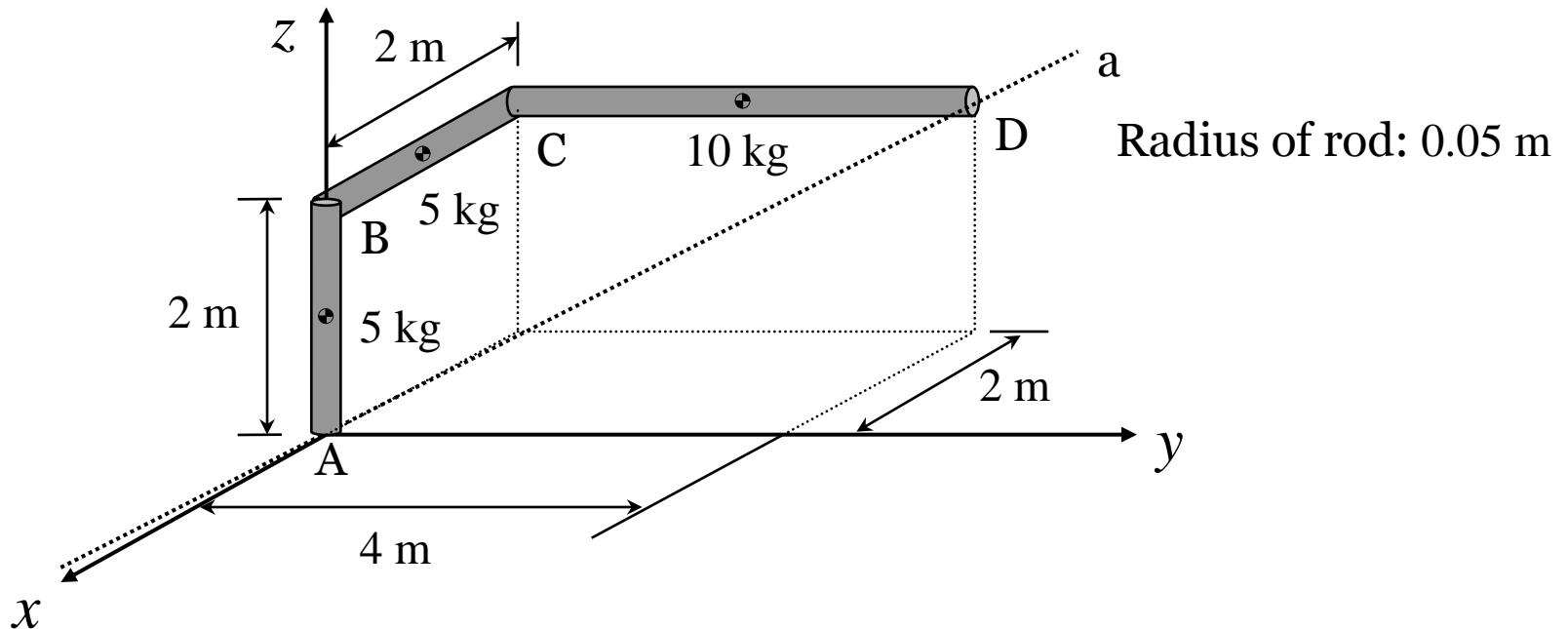
$$I_{Oa} = I_{xx}u_x^2 + I_{yy}u_y^2 + I_{zz}u_z^2 - 2I_{xy}u_xu_y - 2I_{yz}u_yu_z - 2I_{xz}u_xu_z$$

For this calculation the direction cosines u_x , u_y and u_z of the axes must be determined. These numbers specify the cosines of the coordinate direction angles α , β and γ made between the inclined axis and the x , y , z axes respectively.

Rotational dynamics

Single Axis Example

Determine the moment of inertia of the bent arm shown about the Aa axis. The mass of each of the three segments is shown in the figure.

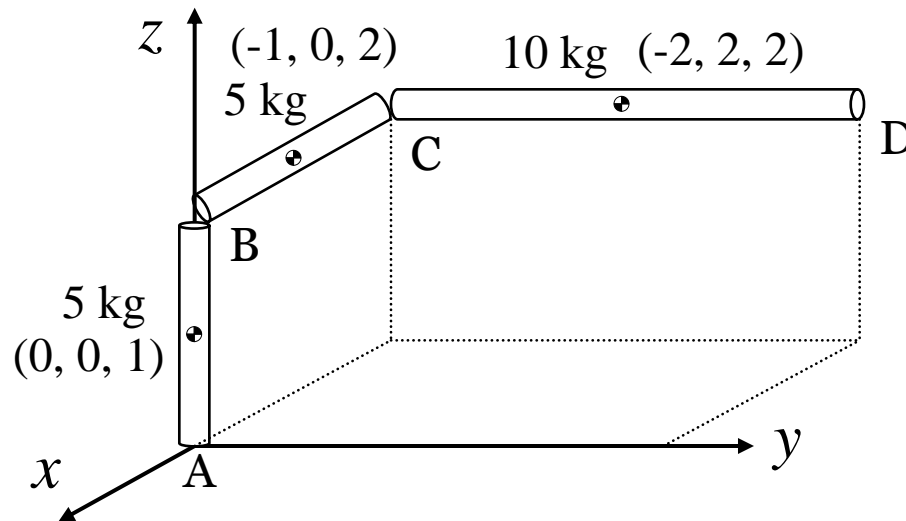


$$I_{Aa} = I_{xx} u_x^2 + I_{yy} u_y^2 + I_{zz} u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{xz} u_x u_z$$

Rotational dynamics

Single Axis Example

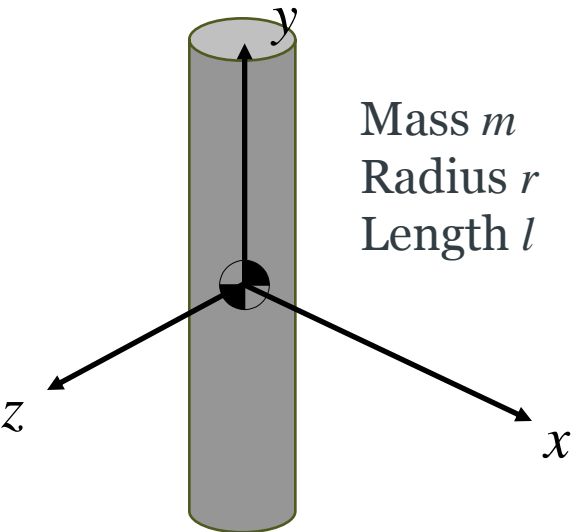
Determine the moment of inertia of the bent rod shown about the Aa axis. The mass of each of the three segments is shown in the figure.



Rotational dynamics

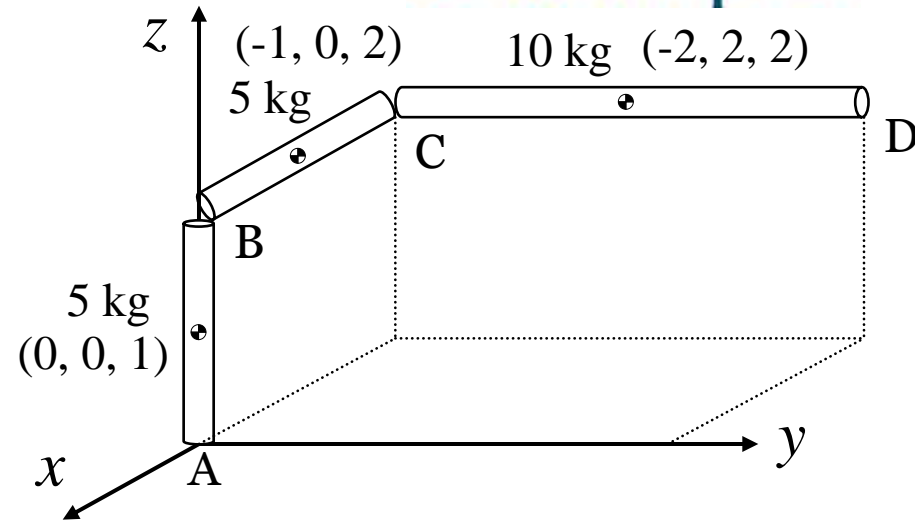
Single Axis Example

Moments and products of inertia of the solid cylinder segments



Products of inertia around CG:

$$I_{xy} = I_{yz} = I_{xz} = 0$$



Moment of inertia around axis running laterally through the CG

$$I_{xx} = I_{zz} = \frac{ml^2}{12}$$

Moment of inertia around axis running axially through the CG

$$I_{yy} = \frac{mr^2}{2}$$

Rotational dynamics

Single Axis Example

Moment of inertia - I_{xx}

$$I_{xx} = \left(I_{xx,1} + m_1 d_{1x}^2 \right) \text{ Segment 1 - AB}$$

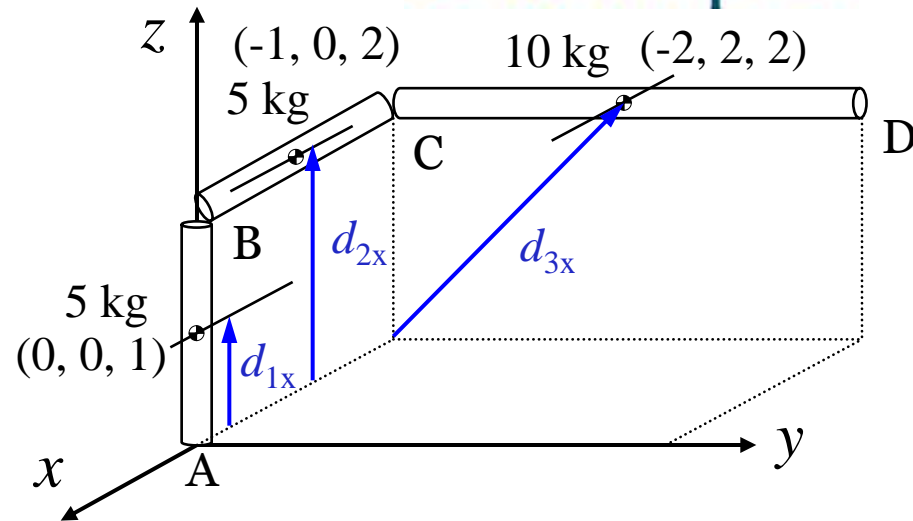
$$+ \left(I_{xx,2} + m_2 d_{2x}^2 \right) \text{ Segment 2 - BC}$$

$$+ \left(I_{xx,3} + m_3 d_{3x}^2 \right) \text{ Segment 3 - CD}$$

$$I_{xx} = \left(\frac{m_1 l_1^2}{12} + m_1 d_{1x}^2 \right) + \left(\frac{m_2 r^2}{2} + m_2 d_{2x}^2 \right) + \left(\frac{m_3 l_3^2}{12} + m_3 d_{3x}^2 \right)$$

$$I_{xx} = \left(\frac{(5)(2)^2}{12} + (5)(1)^2 \right) + \left(\frac{(5)(0.05)^2}{2} + (5)(2)^2 \right) + \left(\frac{(10)(4)^2}{12} + (10)((2)^2 + (2)^2) \right)$$

$$I_{xx} = 6.667 + 20.00625 + 93.333 = 120 \text{ kgm}^2$$



Rotational dynamics

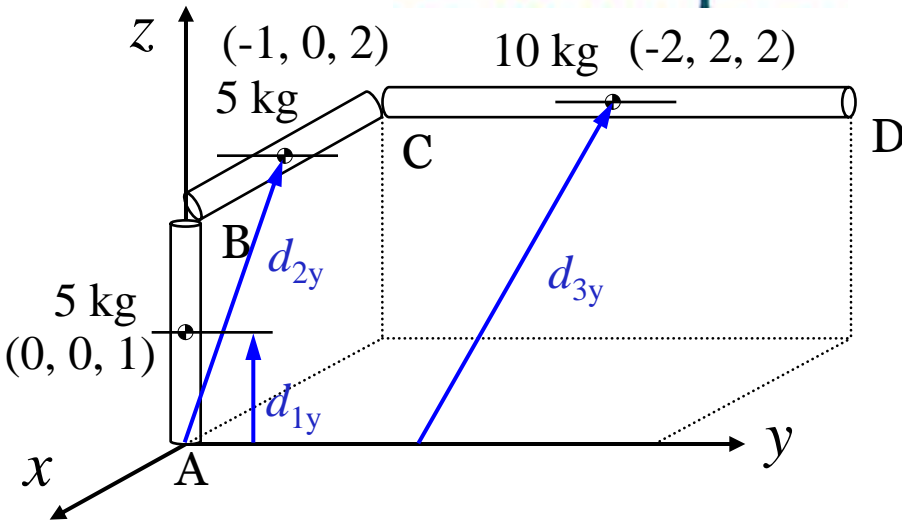
Single Axis Example

Moment of inertia - I_{yy}

$$I_{yy} = \left(\frac{m_1 l_1^2}{12} + m_1 d_{1y}^2 \right) + \left(\frac{m_2 l_2^2}{12} + m_2 d_{2y}^2 \right) + \left(\frac{m_3 r^2}{2} + m_3 d_{3y}^2 \right)$$

$$I_{yy} = \left(\frac{(5)(2)^2}{12} + (5)(1)^2 \right) + \left(\frac{(5)(2)^2}{12} + (5)((-1)^2 + (2)^2) \right) + \left(\frac{(10)(0.05)^2}{2} + (10)((-2)^2 + (2)^2) \right)$$

$$I_{yy} = 6.667 + 26.667 + 80.0125 = 113.35 \text{ kgm}^2$$



Rotational dynamics

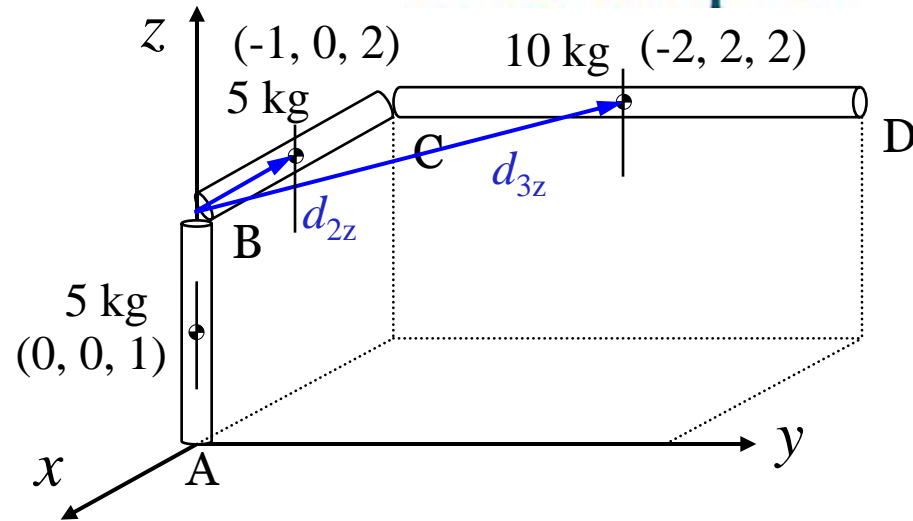
Single Axis Example

Moment of inertia - I_{zz}

$$I_{zz} = \left(\frac{m_1 r^2}{2} \right) + \left(\frac{m_2 l_2^2}{12} + m_2 d_{2z}^2 \right) + \left(\frac{m_3 l_3^2}{12} + m_3 d_{3z}^2 \right)$$

$$I_{zz} = \left(\frac{(5)(0.05)^2}{2} \right) + \left(\frac{(5)(2)^2}{12} + (5)(1)^2 \right) + \left(\frac{(10)(4)^2}{12} + (10)((-2)^2 + (2)^2) \right)$$

$$I_{zz} = 0.00625 + 6.667 + 93.333 = 100 \text{ kgm}^2$$

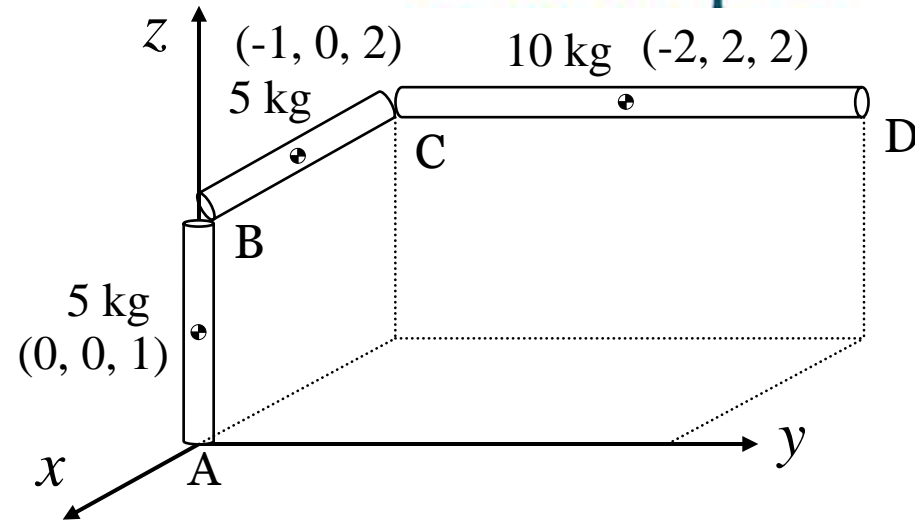


Rotational dynamics

Single Axis Example

Products of inertia

$$I_{xy} = (I_{xy,1} + m_1 x_1 y_1) + (I_{xy,2} + m_2 x_2 y_2) + (I_{xy,3} + m_3 x_3 y_3)$$



As all the elements are uniform solid cylinders their products of inertia around their CG is zero.

$$I_{xy} = (m_1 x_1 y_1) + (m_2 x_2 y_2) + (m_3 x_3 y_3)$$

$$I_{xy} = ((5)(0)(0)) + ((5)(-1)(0)) + ((10)(-2)(2)) = -40 \text{ kgm}^2$$

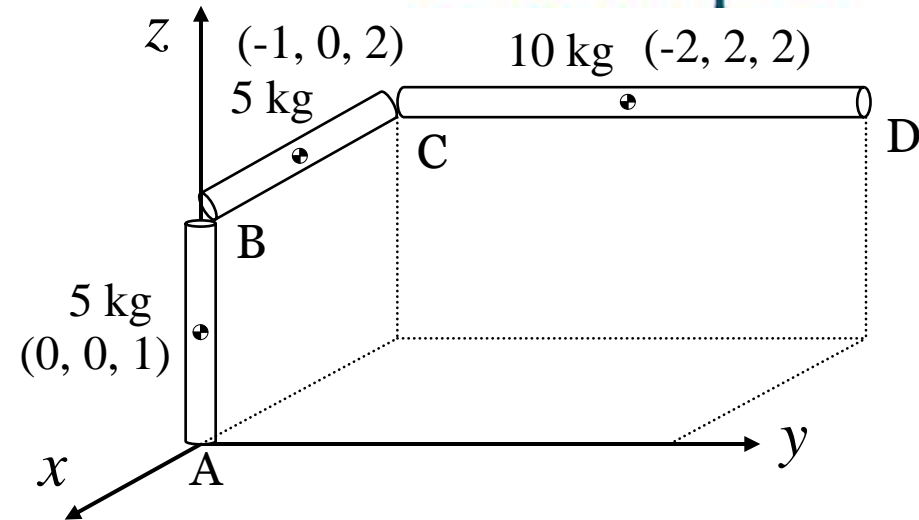
$$I_{yz} = (m_1 y_1 z_1) + (m_2 y_2 z_2) + (m_3 y_3 z_3)$$

$$I_{yz} = ((5)(0)(1)) + ((5)(0)(2)) + ((10)(2)(2)) = 40 \text{ kgm}^2$$

Rotational dynamics

Single Axis Example

Products of inertia



$$I_{xz} = (m_1 x_1 z_1) + (m_2 x_2 z_2) + (m_3 x_3 z_3)$$

$$I_{xz} = ((5)(0)(1)) + ((5)(-1)(2)) + ((10)(-2)(2)) = -50 \text{ kgm}^2$$

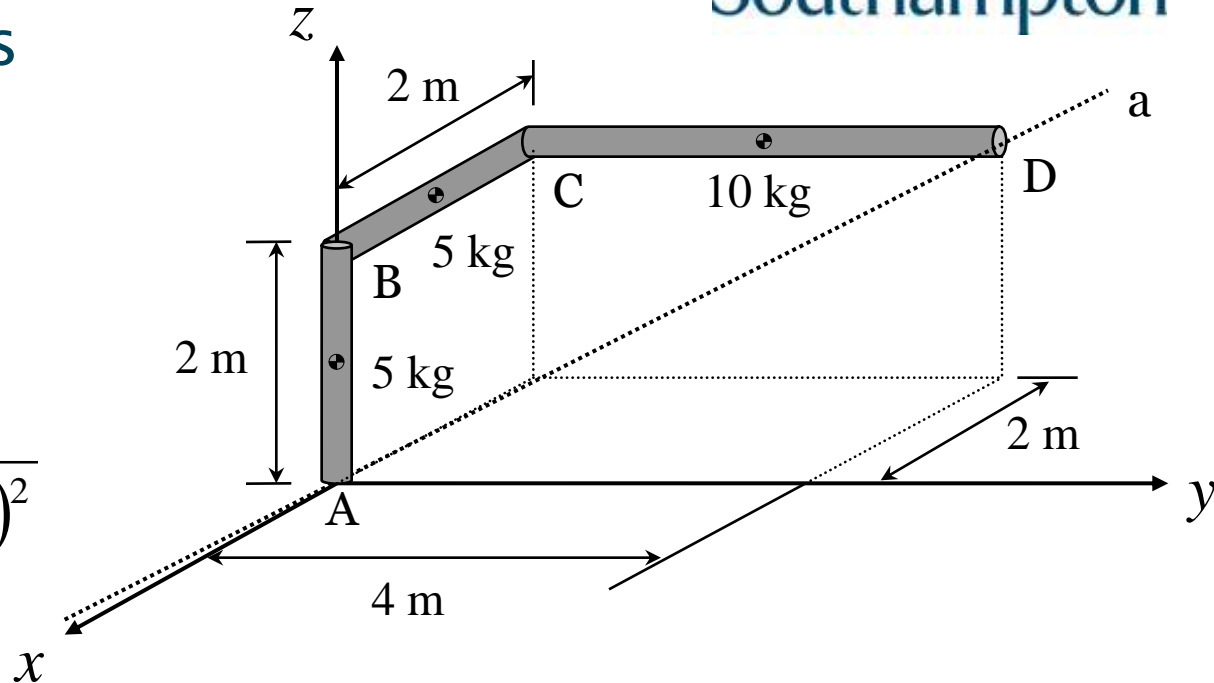
Rotational dynamics

Single Axis Example

Need to determine the direction cosines:

$$\mathbf{r}_{Aa} = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

$$|\mathbf{r}_{Aa}| = \sqrt{(-2)^2 + (4)^2 + (2)^2} = 4.899$$



Unit vector in axis Aa:

$$\mathbf{u}_{Aa} = \frac{\mathbf{r}_{Aa}}{|\mathbf{r}_{Aa}|} = \frac{-2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}}{4.899} = -0.408\mathbf{i} + 0.816\mathbf{j} + 0.408\mathbf{k}$$

$$u_x = -0.408 \quad u_y = 0.816 \quad u_z = 0.408$$

Rotational dynamics

Single Axis Example:

$$I_{Aa} = I_{xx}u_x^2 + I_{yy}u_y^2 + I_{zz}u_z^2 - 2I_{xy}u_xu_y - 2I_{yz}u_yu_z - 2I_{xz}u_xu_z$$

Moments of inertia

$$I_{xx} = 120 \text{ kgm}^2 \quad I_{yy} = 113.35 \text{ kgm}^2 \quad I_{zz} = 100 \text{ kgm}^2$$

Products of inertia

$$I_{xy} = -40 \text{ kgm}^2 \quad I_{yz} = 40 \text{ kgm}^2 \quad I_{xz} = -50 \text{ kgm}^2$$

Direction cosines

$$u_x = -0.408 \quad u_y = 0.816 \quad u_z = 0.408$$

$$I_{Aa} = (120)(-0.408)^2 + (113.35)(0.816)^2 + (100)(0.408)^2 \\ - 2(-40)(-0.408)(0.816) - 2(40)(0.816)(0.408) \\ - 2(-50)(-0.408)(0.408)$$

$$I_{Aa} = 42.2 \text{ kgm}^2$$

Rotational dynamics

The inertia matrix



$[I]$ is an important quantity when sizing up the control system inputs for any vehicle.

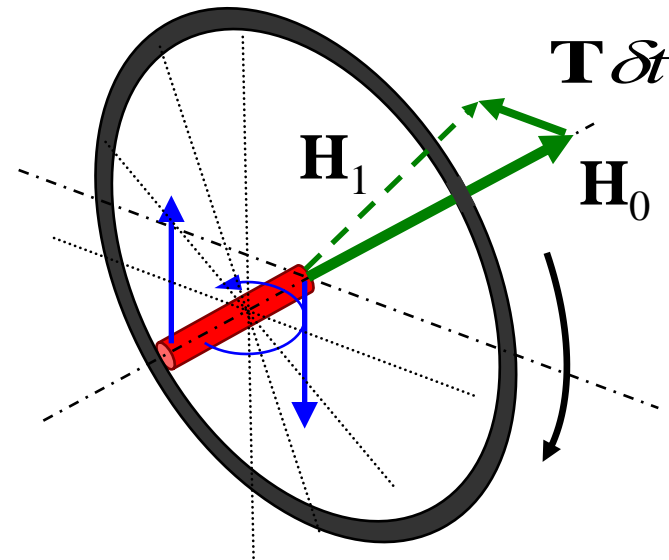
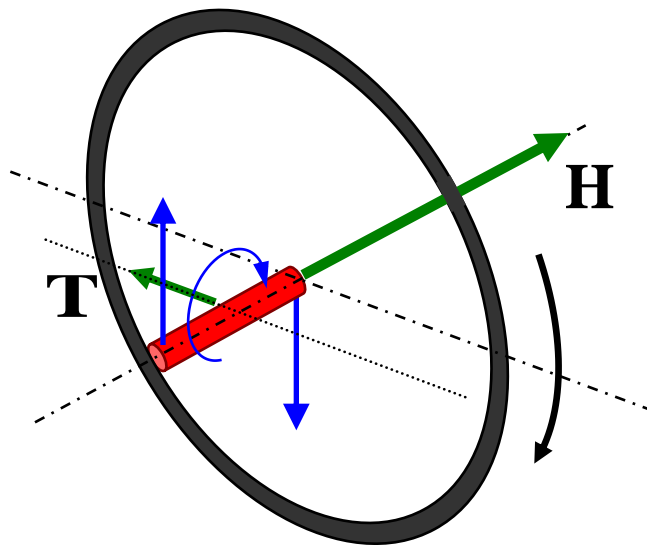
Rotational dynamics

Properties of rotational motion – Gyroscopic Precession



Rotational dynamics

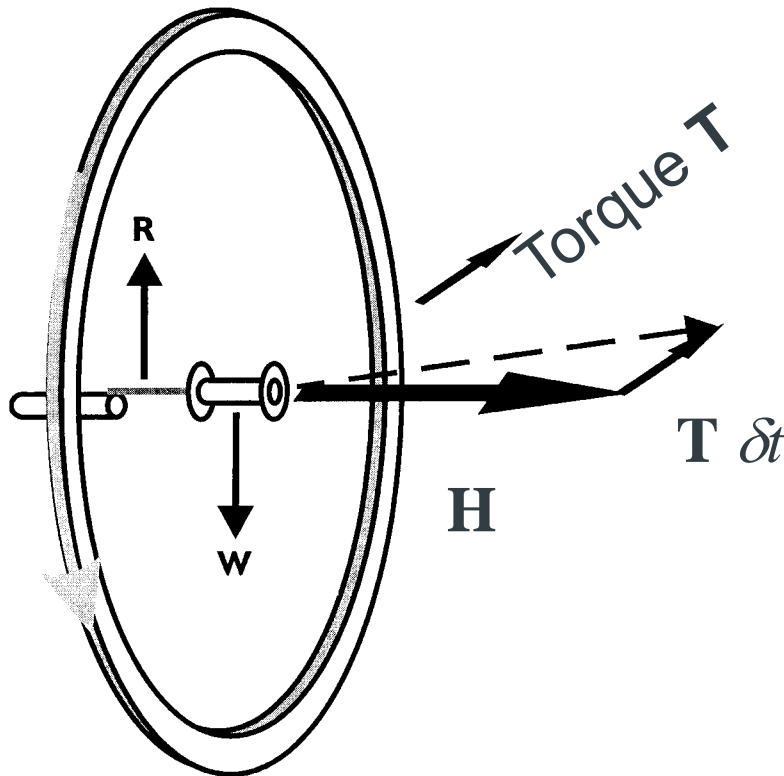
Properties of rotational motion – Gyroscopic Precession



The rotational displacement occurs 90 degrees later in the direction of rotation.

Rotational dynamics

Properties of rotational motion – Gyroscopic Precession



Mass	$\approx 2 \text{ kg}$
Angular rate ω	$\approx 20 \text{ rads/s}$
Moment of inertia	$\approx 0.1 \text{ kg.m}^2$
Angular momentum,	
	$H (= I \omega) = 2 \text{ kg m}^2/\text{s}$

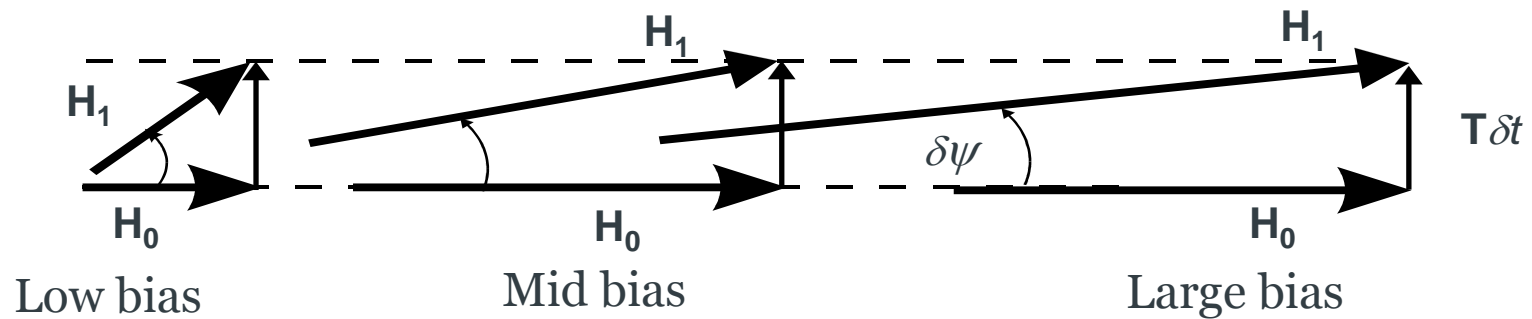
Rotational dynamics

Properties of rotational motion

Momentum Bias/Gyroscopic rigidity

Momentum reduces sensitivity to torque

During δt , the momentum changes direction $\delta\psi$ from \mathbf{H}_0 to \mathbf{H}_1



Rotational dynamics

Use of Momentum Bias

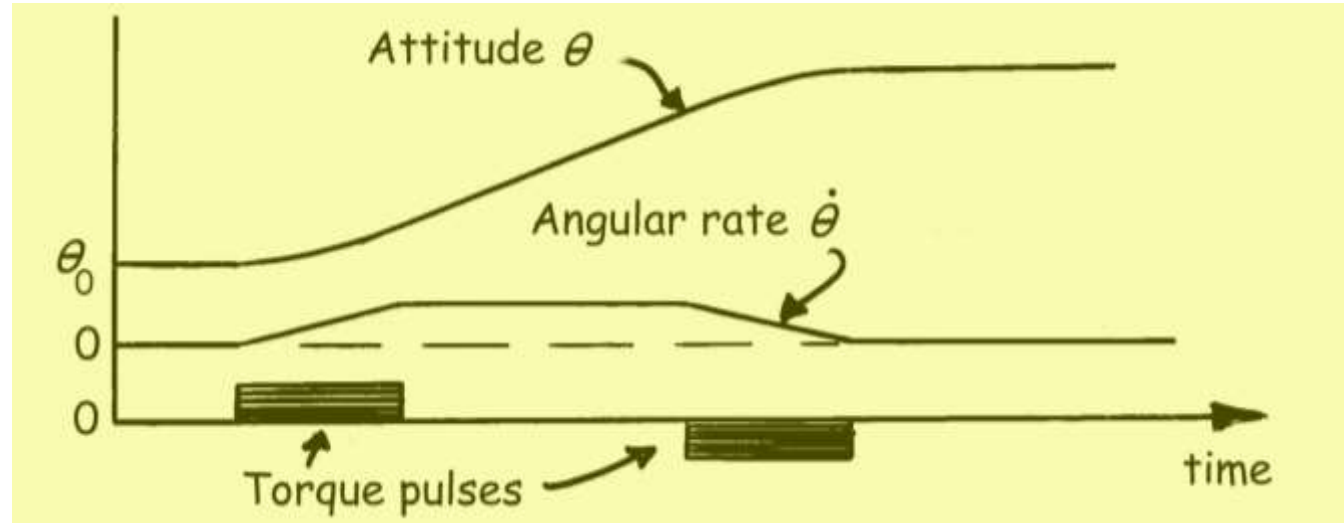
Momentum bias is a method commonly used to provide inherent stability. However, there are consequences of doing so...

- to use momentum bias, it is desirable that one body axis of the spacecraft remains invariantly pointing (usually perpendicular to the orbit plane)
- bias introduces an oscillatory **nutation mode**
- a system with bias will have different torque responses

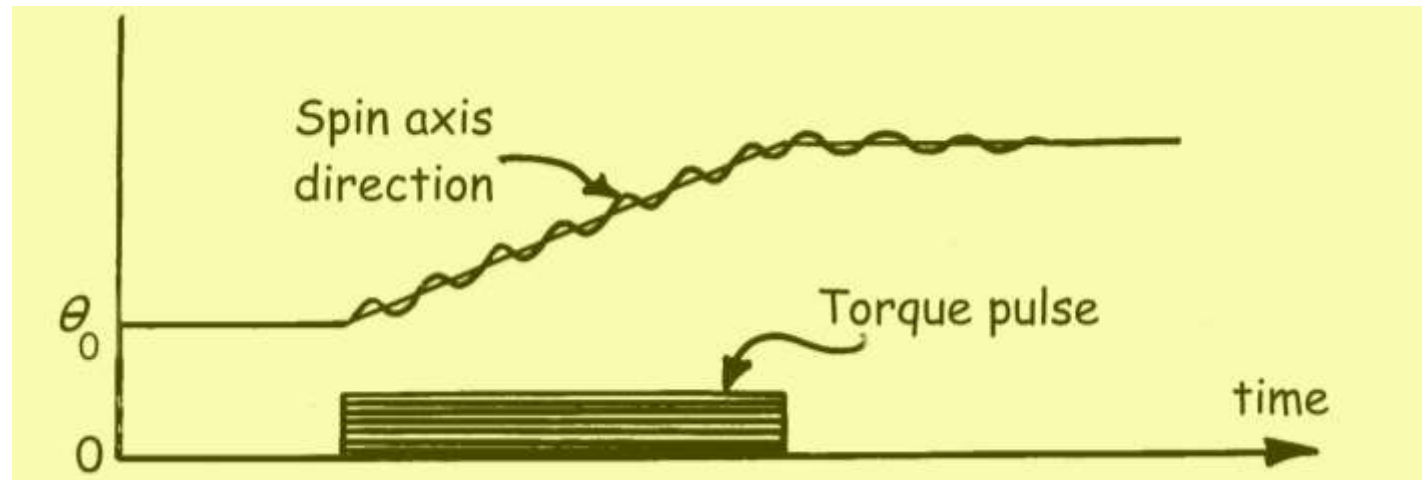
Rotational dynamics

Use of Momentum Bias – Torque responses

Torque response without bias

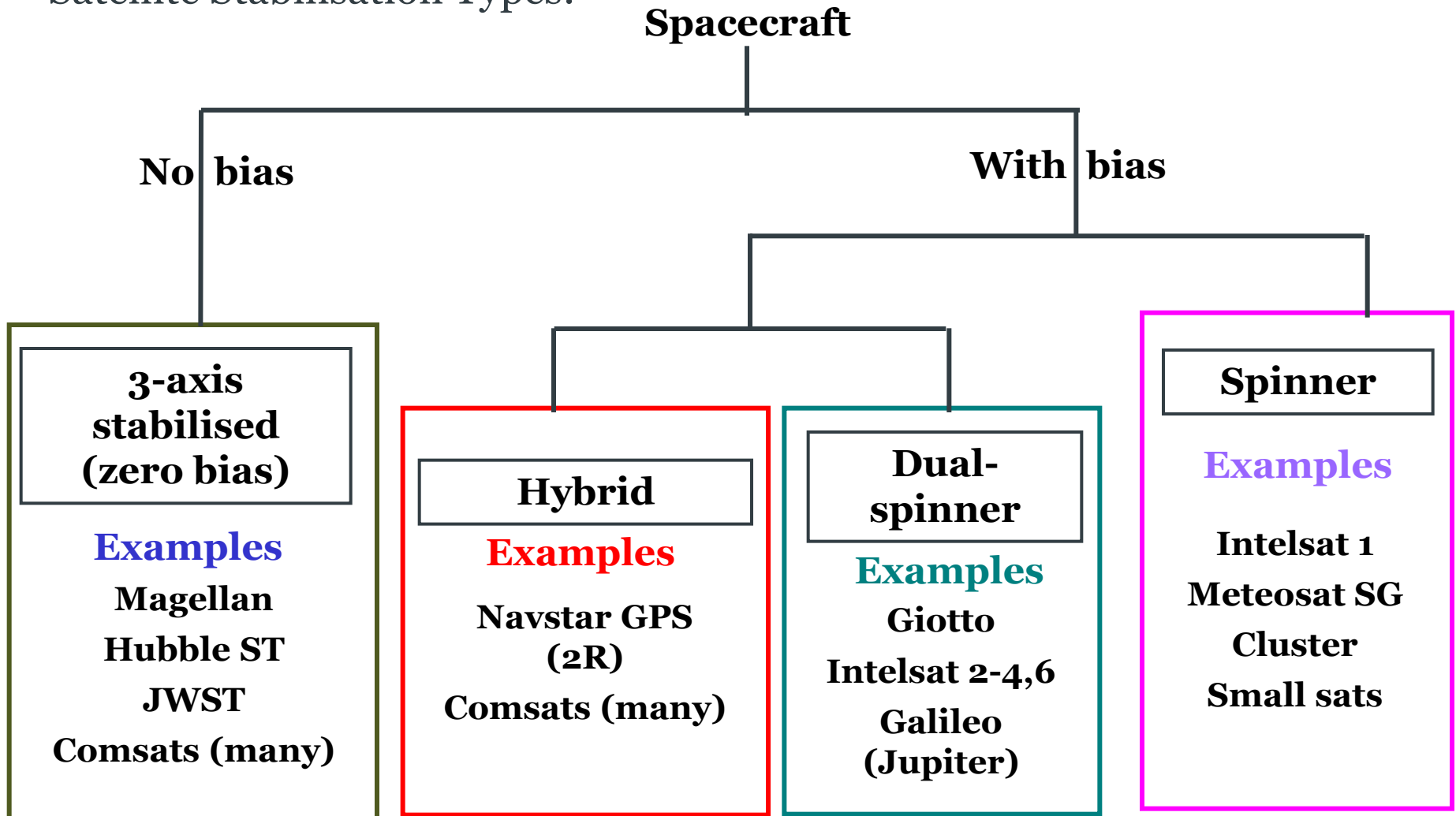


Torque response with bias



Rotational dynamics

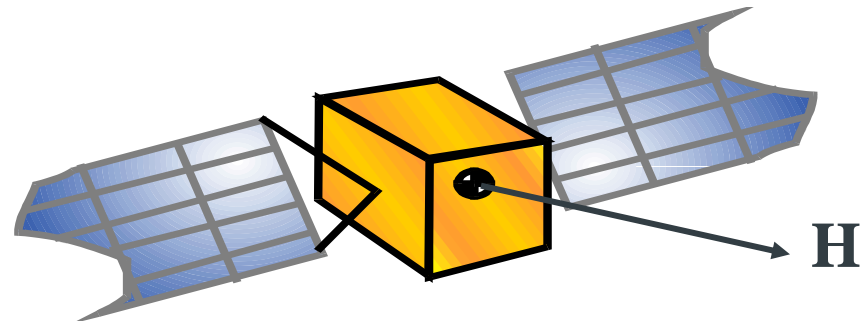
Satellite Stabilisation Types:



Rotational dynamics

Momentum Management

- The ACS must ‘manage’ the momentum \mathbf{H} of the spacecraft using control torquers to do so.



- This can be achieved using the principles of:
Conservation of momentum – the storage/transfer of momentum

$$(\Sigma \mathbf{T}_{\text{ext}} = \mathbf{0} \Rightarrow \text{Momentum } \mathbf{H} \text{ is constant})$$

Newton's second law – by applying a torque to the satellite

$$(\Sigma \mathbf{T}_{\text{ext}} \neq \mathbf{0} \Rightarrow \text{Momentum } \mathbf{H} \text{ changes in magnitude/direction})$$

Rotational dynamics

Categories of Torques

External torques

- due to reactions with the environment
- i.e. a torque is applied which *changes* the total angular momentum of the satellite

Internal torques

- due to reactions between two parts of the spacecraft
- by definition no external torque is applied, therefore the total angular momentum is conserved

Rotational dynamics

External torques/torquers

Naturally occurring (disturbance) torques:

- Aerodynamic < ~ 500 km
- Magnetic ~ 500 – 35,000 km
- Solar radiation > ~ 600 – 700 km
- Gravity gradient ~ 500 – 10,000 km
(Thrust misalignment)

Controllable external torquers

- Gas jets all heights
- Magnetorquers up to synchronous
- Adjustable geometry

Rotational dynamics

Internal torques/torquers

Internal disturbance torques:

- Mechanisms – deploying solar arrays
- Fuel movement ('slosh')
- Astronaut movement

Controllable internal torquers (momentum stores)

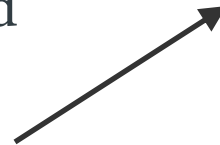
- Reaction wheels
- Momentum wheels

As the ACS must 'manage' the momentum H of the spacecraft therefore one type of external torquer *must* be carried.

ACS for Rendezvous

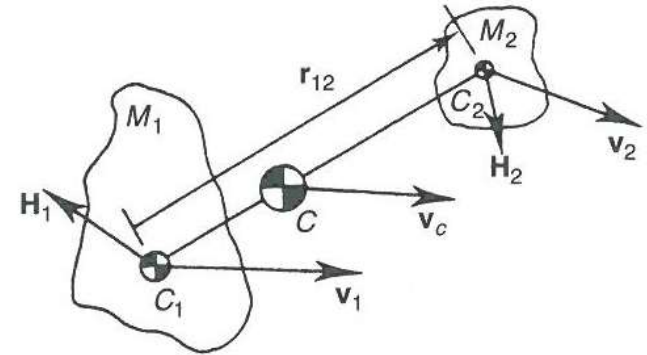
Prime purposes:

- To achieve the pointing requirements of the payload
the capture and control the proposed target
- To achieve the pointing requirements for 'house-keeping'
 - in all phases of the mission
 - e.g. Power-raising - Sun-pointing
 - Communications - Earth-pointing
 - Thermal - Deep space
 - Orbit change thruster - as required
- To manage the (angular) momentum
 - The inertia matrix
 - Choice of external torquers
 - The use of spinning systems



Rendezvous

Multiple Rigid Bodies



For the system: during a collision/separation:

- their total absolute linear momentum remains constant

$$M\mathbf{v}_C = M_1\mathbf{v}_1 + M_2\mathbf{v}_2 \quad \text{remains constant}$$

- their angular momenta referred to C remains constant, so

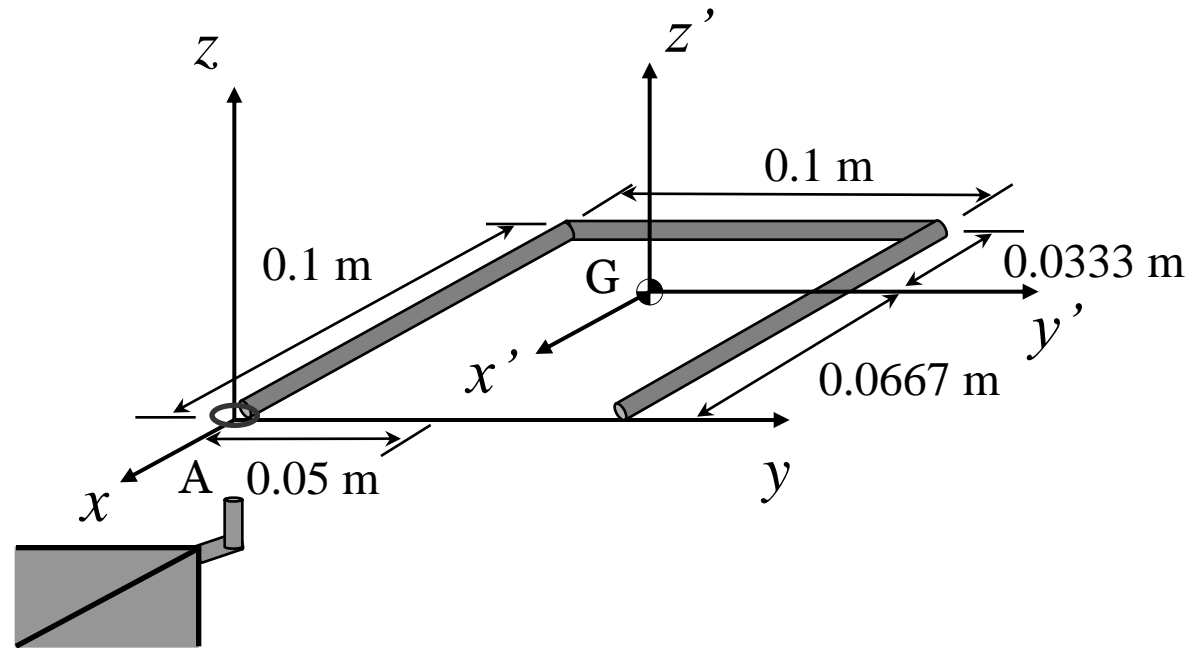
$$\mathbf{H}_C = \left(\frac{M_1 M_2}{M} \right) (\mathbf{r}_{12} \times \mathbf{v}_{12}) + \mathbf{H}_1 + \mathbf{H}_2 \quad \text{remains constant}$$

where \mathbf{r}_{12} , \mathbf{v}_{12} are the position and velocity vectors of C_2 relative to C_1

Rendezvous

Conservation of Angular Momentum Example

The rod has a total mass of 0.6 kg. Determine its angular velocity just after the end A falls on to the hook. The hook provides a permanent connection for the rod (i.e. it has a spring lock mechanism).



Just before striking the hook the rod is falling downward with a speed $V_1 = 1$ m/s. The rod also has the following moments of inertia about its CG position:

$$I_{x'x'} = 1.2 \times 10^{-3} \text{ kgm}^2$$

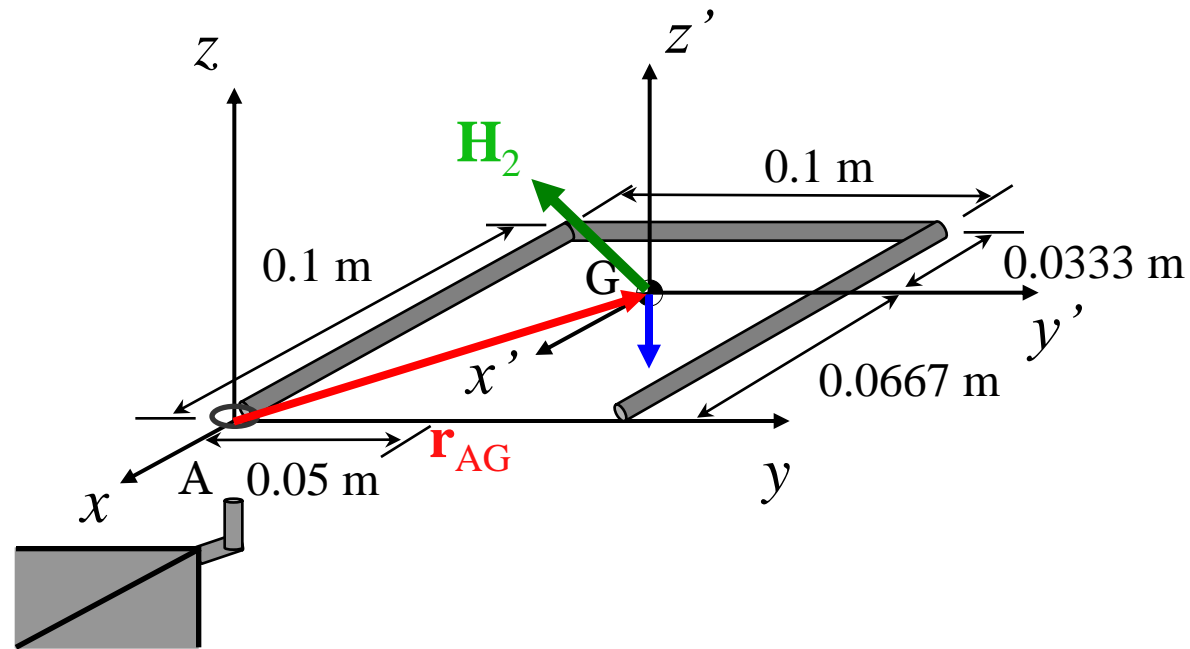
$$I_{y'y'} = 0.7 \times 10^{-3} \text{ kgm}^2$$

$$I_{z'z'} = 1.8 \times 10^{-3} \text{ kgm}^2$$

Rendezvous

Conservation of Angular Momentum Example

An impulsive force acts from the hook to change the momentum of the rod. However the angular momentum of the rod is conserved about point A since the moment arm of the impulsive force is zero.



At first time step before impact:

$$\mathbf{r}_{AG} \times m(\mathbf{v}_1) = \mathbf{r}_{AG} \times m(\mathbf{v}_2) + \mathbf{H}_2$$

$$\mathbf{r}_{AG} = -0.0667\mathbf{i} + 0.5\mathbf{j}$$

$$\mathbf{v}_1 = -1\mathbf{k}$$

At second time step after impact:

$$\mathbf{H}_2 = \mathbf{I}\boldsymbol{\omega}$$

$$\mathbf{H}_2 = I_{x'x'}\omega_x\mathbf{i} + I_{y'y'}\omega_y\mathbf{j} + I_{z'z'}\omega_z\mathbf{k}$$

Rendezvous

Conservation of Angular Momentum Example

$$\begin{aligned} \{-0.0667\mathbf{i} + 0.5\mathbf{j}\} \times \{-0.6\mathbf{k}\} &= \{-0.0667\mathbf{i} + 0.5\mathbf{j}\} \times \{-0.6v_2\mathbf{k}\} \\ &+ \left\{ (1.2 \times 10^{-3})\omega_x\mathbf{i} + (0.7 \times 10^{-3})\omega_y\mathbf{j} + (1.8 \times 10^{-3})\omega_z\mathbf{k} \right\} \\ -0.03\mathbf{i} - 0.04002\mathbf{j} &= -0.03v_2\mathbf{i} - 0.04002v_2\mathbf{j} \\ &+ \left\{ (1.2 \times 10^{-3})\omega_x\mathbf{i} + (0.7 \times 10^{-3})\omega_y\mathbf{j} + (1.8 \times 10^{-3})\omega_z\mathbf{k} \right\} \end{aligned}$$

Equating i, j and k components:

$$\begin{aligned} -0.03 &= -0.03v_2 + (1.2 \times 10^{-3})\omega_x \\ -0.04002 &= -0.04002v_2 + (0.7 \times 10^{-3})\omega_y \\ 0 &= (1.8 \times 10^{-3})\omega_z \quad \rightarrow \omega_z = 0 \end{aligned}$$

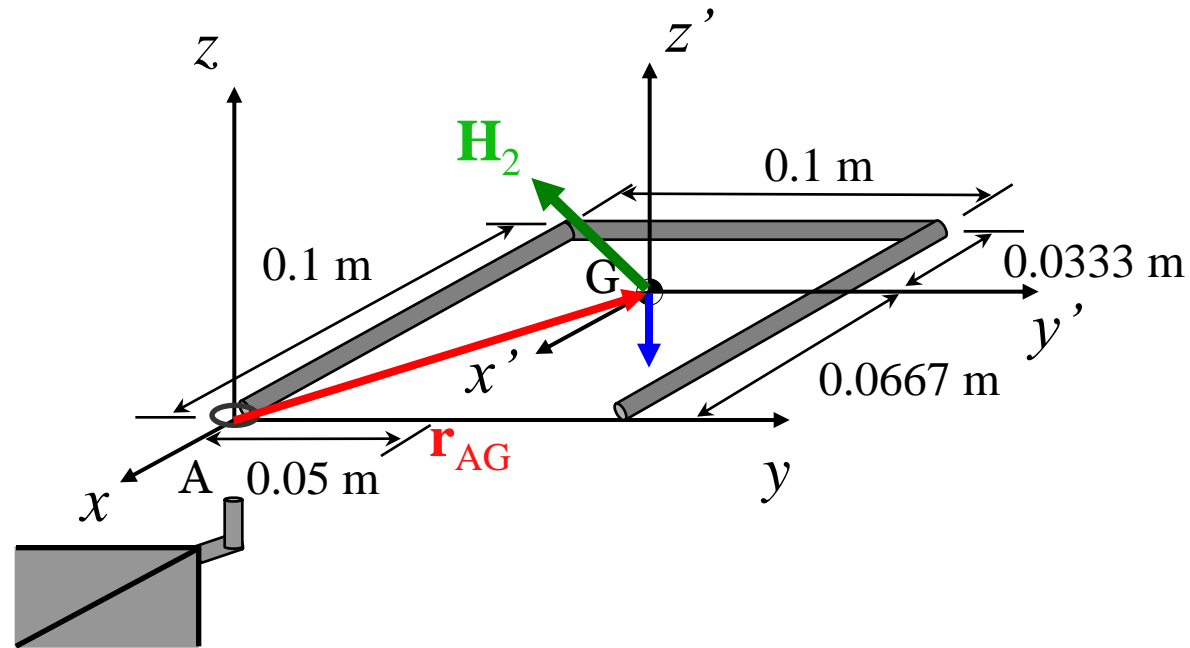
However, we still have 2 equations and 3 unknowns...

Rendezvous

Conservation of Angular Momentum Example

After impact the mass will fall in a circular arc around A so:

$$\mathbf{v}_2 = \boldsymbol{\omega} \times \mathbf{r}_{AG}$$



$$\begin{aligned} -v_2 \mathbf{k} &= \{\omega_x \mathbf{i} + \omega_y \mathbf{j}\} \times \{-0.0667 \mathbf{i} + 0.05 \mathbf{j}\} \\ &= (0.05 \omega_x + 0.0667 \omega_y) \mathbf{k} \end{aligned}$$

$$-v_2 = 0.05 \omega_x + 0.0667 \omega_y$$

Rendezvous

Conservation of Angular Momentum Example

So the equations are:

$$-0.03 = -0.03v_2 + (1.2 \times 10^{-3})\omega_x$$

$$-0.04002 = -0.04002v_2 + (0.7 \times 10^{-3})\omega_y$$

$$0 = v_2 + 0.05\omega_x + 0.0667\omega_y$$

Which can be solved to give:

$$v_2 = 0.8351 \text{ m/s}$$

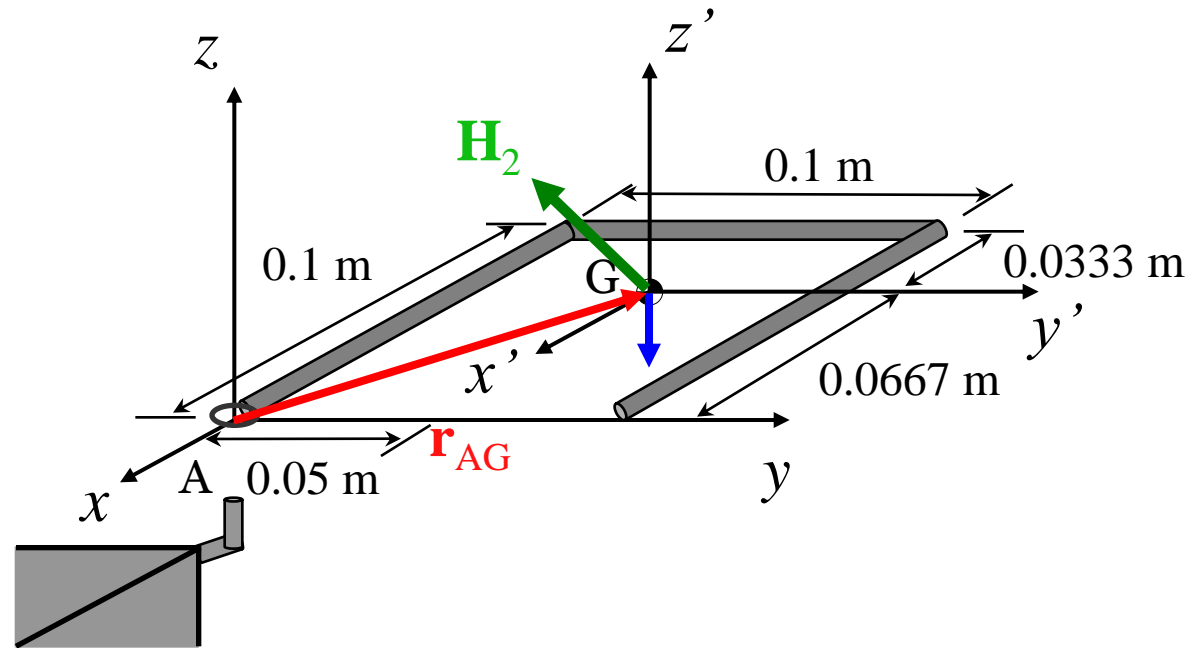
$$\boldsymbol{\omega} = -4.12\mathbf{i} - 9.43\mathbf{j} \text{ rad/s}$$

Rendezvous

Conservation of Angular
Momentum Example

$$v_2 = 0.8351 \text{ m/s}$$

$$\boldsymbol{\omega} = -4.12\mathbf{i} - 9.43\mathbf{j} \text{ rad/s}$$



Rendezvous

Critical parameters for ACS

Target selection

- size
- orbit

Target properties

- total mass (range?)
- Centre of gravity position
- inertia matrix (mass distribution)
- tumbling?
 - angular velocities of tumbling
 - total angular momentum

Rendezvous

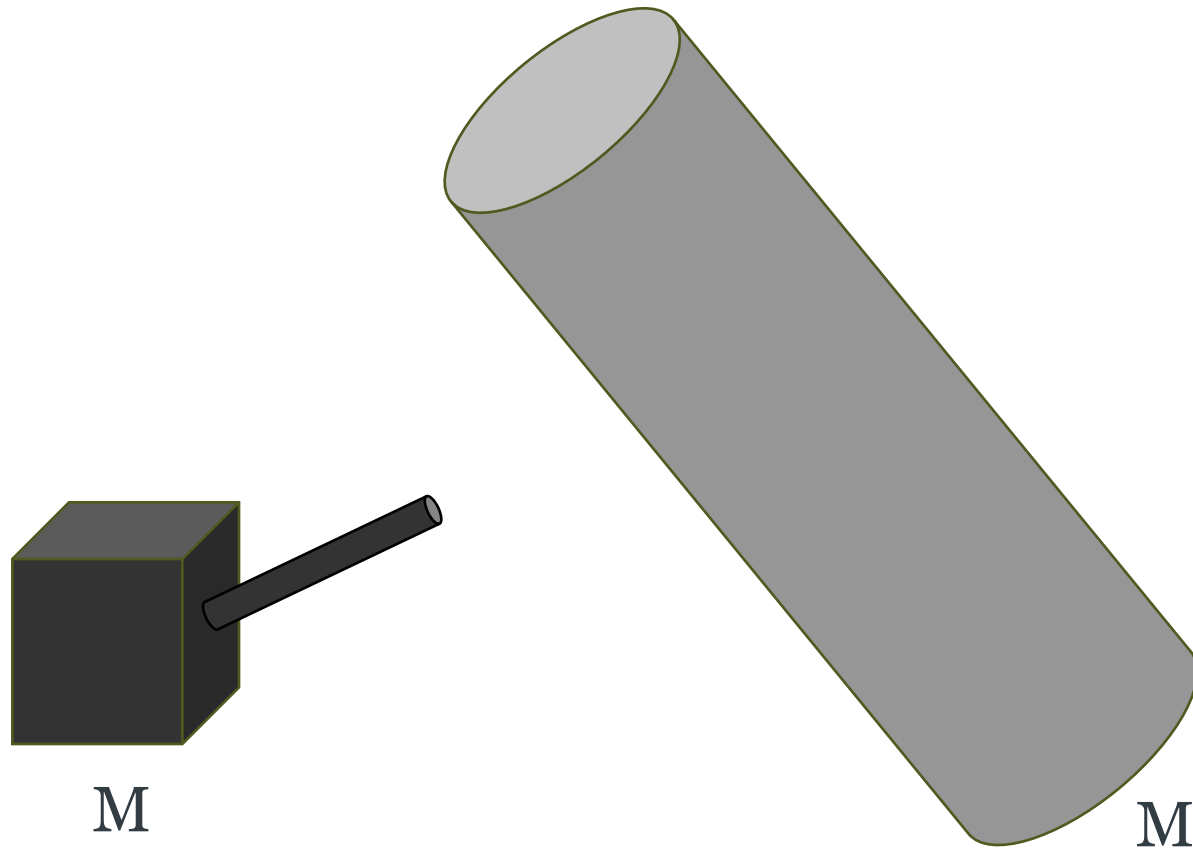
The problem for the ACS system:



Would like to know the combined CG and the inertia matrix to ensure we have enough command authority to control the combined system

Rendezvous

The problem for the ACS system:



Rendezvous

De-tumbling options

To de-tumble a target object the total angular momentum of the combined system has to be reduced

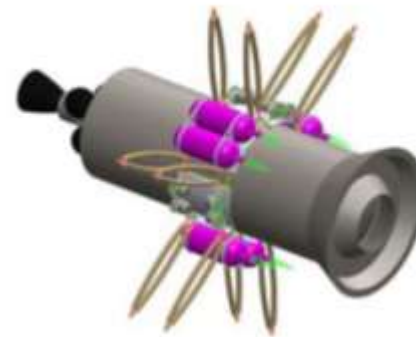
Non-contact

Exhaust products directed onto the tumbling object.

Contact

Use external torquers to control the tumbling motion – thrusters, variable area geometry?

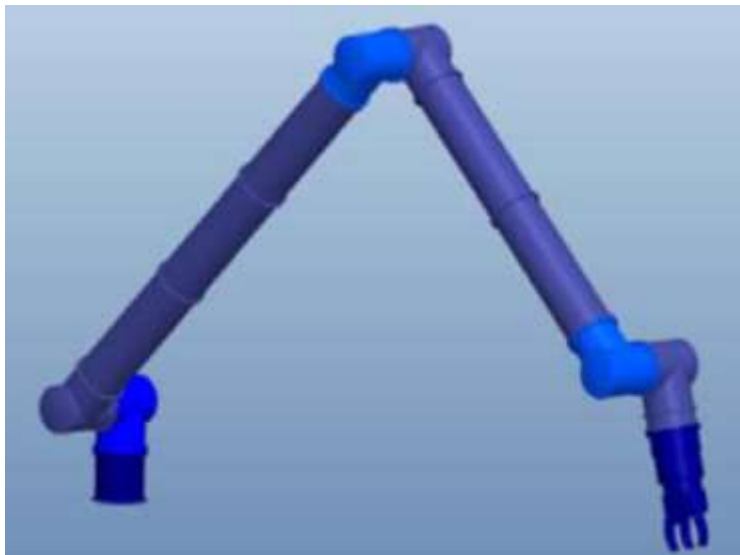
Retractable ‘gate’ concept



Rendezvous

Prime solutions (from Astrium's perspective)

Robotic Arm



Courtesy of DLR

Net solutions



ROGER net system



Conclusion

Rotational Dynamics and Attitude Control

The effective design of the AOCS subsystem on the chaser spacecraft is critical to the success of any ADR mission.

The requirements for the AOCS design is significantly more challenging than any standard space mission.

The larger the target object, the greater the challenge.

It involves an understanding of the combined three dimensional inertial properties, angular momentum and rotational dynamics.